Three-dimensional analyses of electric currents and pressure anisotropies in the plasma sheet

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[1] Long-term averaged three-dimensional (3-D) database magnetotail models have been created for many plasma and field parameters. A simple modeling technique that was used in earlier work is compared with a new more comprehensive procedure. Good agreement between the two methods was found when both produced stable results. The simpler method generated reliable models from any of the available data sets, while the more complex method did not. It is shown how analyses of the models can determine plasma and field parameters that are too small to be measured directly. The magnetic field data was used to calculate average electric currents flowing in the x and y directions. Full 3-D distributions of \( j_x \) and \( j_y \) were determined from the models even though these currents were too small to measure directly with adequate accuracy. Changes of the electron and ion pressure anisotropies as a function of distance along an average magnetic field line then were analyzed. It was concluded that the electron anisotropy was created by an electric field with a parallel component near the neutral sheet. The parallel electric field is small but is required to maintain charge neutrality in the region containing guiding center electron and nonguiding center ion orbits. In addition to creating anisotropic electron distributions the presence of this parallel electric field violates the ideal MHD assumptions near the neutral sheet. The ion anisotropy suggests that the net effect of chaotic ion motion near the neutral sheet is to create a weak source cone distribution superimposed on a denser isotropic component.

1. Introduction

[2] Techniques were developed to create three-dimensional (3-D) database models of the average values of plasma and field parameters within the plasma sheet. The basic procedure is described by Kaufmann et al. [2001, hereinafter referred to as paper 1]. The models are based on data from the Geotail satellite and cover the region between 10 and 30 \( R_E \). The measurements were made by the Comprehensive Plasma Instrumentation (CPI) [Frank et al., 1994] and the magnetic field detector (MGF) [Kokubun et al., 1994].

[3] The 3-D aspect of the models is the feature that distinguishes paper 1 and the present work from most other plasma sheet studies. Essentially all magnetic field models and many magnetosphere simulations are 2-D, but database models of plasma parameters have primarily been 2-D. The value of a 3-D model is that it permits one to calculate derivatives of all the plasma and field parameters that appear in the fluid equations. A 3-D model also is needed to see how parameters change as one moves along an average magnetic field line. For example, calculating the average parallel current using all data with \( \beta > 1 \) at a fixed \( x-y \) location such as \( x = -10 R_E, \ y = 0 \) would involve averaging over sections of field lines that reach the equator between \( x = -10 R_E \) and \( x = -20 R_E \). Some of these field lines could carry region 1 current, some no parallel current, and some region 2 current. Interpretation of the average of these currents therefore would be difficult.

[4] The 2-D models that have been developed typically used a parameter such as \( B_x < B_x^{0} \) or the ordinary plasma \( \beta > \beta_0 \) to select all plasma sheet data, where \( B_x^{0} \) and \( \beta_0 \) are values representative of the outer edge of the plasma sheet. For example, Baumjohann et al. [1989] used \( B_y < 15 \) nT to separate inner central plasma sheet (CPS) from outer CPS data, where \( B_y^{0} = B_x^{0} + B_y^{0} \). Angelopoulos et al. [1994] used the ion beta \( \beta_i > 0.5 \) to define the inner plasma sheet, and Huang and Frank [1986] required both density \( n > 0.01 \) \( \text{cm}^{-3} \) and temperature \( kT > 200 \) eV to define the central plasma sheet.

[5] A few other studies used \( B_x, \beta \), or a related parameter to arrange all data according to the estimated distance from
the neutral sheet. Bowling and Wolf [1974] sorted data from six orbits by $B_x$ to estimate that the plasma sheet was between 2 and 3 $R_E$ thick at $x = -30$ $R_E$. Troschichev et al. [1999] sorted by $B_y$ to study convection and other plasma parameters in the distant tail. Kistler et al. [1993] used the ratio of the local plasma pressure to the total plasma plus field pressure to order data according to estimated distance from the neutral sheet.

The modeling procedure used here begins by placing the plasma and field data into $3 \times 3$ or $6 \times 6$ $R_E$ x-y boxes using aberrated GSM coordinates. The measured parameters have relatively long scale lengths of 5 to 10 $R_E$ in both the x and y directions. As a result, satellite trajectory information is sufficient to determine the x-y box into which each data point should be placed.

The scale lengths in the z direction are much smaller than those in the x and y directions. In addition, the plasma sheet often moves up and down in the z direction with an amplitude of several Earth radii and a period of $\sim 10$ min. A number of papers have calculated plasma sheet thicknesses by assuming that the electric current structure is fixed for the time required for the plasma sheet to move over one or two satellites. McComas et al. [1986]; Sonny et al. [1994]; Zhou et al. [1997]; and Sergeev et al. [1998] used this method to derive consistent plasma sheet thicknesses for a few orbits.

Since the z dependence of parameters in the thin region of nonadiabatic ion orbits is of particular interest to the present study, trajectory information is not adequate to determine the distance from the satellite to the neutral sheet. We used a technique that is somewhat different from those used in the papers referenced above to estimate z locations. The technique selected here is useful when treating large data sets rather than individual orbits. Data within each x-y box first were sorted according to either the ordinary plasma component of the field, $B_x$, or the magnetic field pressure based only upon the x component of the field, $B_x^2/2\mu_0$. Each of these two sorting methods is useful for certain studies. The models shown in this paper used $\beta_b$ as the sorting parameter that is, by definition, infinite at the neutral sheet. The assumption made is that $\beta_b$ decreases monotonically as one moves to the outer plasma sheet. The $\beta_b$ parameter was picked because it best identifies the neutral sheet everywhere in the tail. The ordinary $\beta$ parameter also is assumed to be largest near the neutral sheet at any given x-y. However, $\beta$ becomes very large (e.g., $\beta > 30$) only when the tail is highly stretched and when the current sheet is very thin. The use of $\beta$ therefore would be appropriate for studies of very thin current sheets.

The critical step in the modeling procedure involves converting from the nonorthogonal ($x, y, \beta_b$) coordinate system to the orthogonal ($x, y, z$) system. This conversion is needed because a study of almost any physically significant aspect of the plasma sheet requires derivatives of the measured parameters such as $\nabla \times \mathbf{B}$ or $\nabla \cdot \mathbf{P}$, where $\mathbf{P}$ is the pressure tensor. These partial derivatives must be evaluated in an orthogonal coordinate system. The conversion procedure described in paper 1 makes use of the momentum equation to convert from $\beta_b$ to z. Including the average plasma acceleration as an inertial force, the z thickness of each ($x, y, \beta_b$) box is adjusted so that the total long-term averaged net force on the box is zero. The procedure has proven to be very reliable. Results are similar when data from any of the four years of available measurements are analyzed using either $3 \times 3$ or $6 \times 6$ $R_E$ x-y spacing. Results presented here combined all 4 years of data to produce the smoothest possible model.

The conceptual problem with the technique described above is that only a single height or z thickness can be calculated for each ($x, y, \beta_b$) box. These boxes therefore are treated to some extent as rectangular objects. The analysis method described in the appendix uses boxes with more complex shapes. Instead of calculating a single thickness for a box, the heights of the four edges of each box are determined separately. Some symmetry assumptions are required so that all edge lengths can be evaluated. Although the method achieves force balance for each of the nonrectangular boxes, the results are too sensitive to noise, errors, and fluctuations in the data to be generally useful. When $6 \times 6$ $R_E$ x-y boxes are used with the combined 4 years of data then the two methods agree throughout much of the CPS. This agreement between the two methods provides some confidence that the 3-D models are realistic. The method described in the appendix becomes unstable even near the neutral sheet when less data or smaller boxes are used.

The present paper contains studies of electric currents and of pressure anisotropies. The most significant results of these particular studies are that analyses of the models can determine the values of some quantities that are too small to be measured directly. Information is obtained about the 3-D distributions of currents, about parallel electric fields, and about some consequences of chaotic ion motion near the neutral sheet. Section 2 describes the analysis of electric currents flowing in the x and y directions. The z current density is so small that we were unable to obtain meaningful estimates using direct electron and ion measurements (paper 1). An analysis of parallel currents can also be carried out but is more complex and is not included in this paper. The average magnetic field configuration is shown in section 3. This is used to study changes in the pressure anisotropies $P_b/P_\perp$ for electrons and ions as one moves away from the neutral sheet along average magnetic field lines. This part of the analysis provides some information about parallel electric fields and the consequences of chaotic motion near the neutral sheet. Section 4 is a summary of the results.

2. Electric Currents

2.1. Box Heights

Figure 1 shows the factors used in this paper to convert from $\beta_b$ to z for each x-y box. Each panel relates z to $\beta_b$ for one x-y box whose location is given by the labels at the top and right side of Figure 1. The derivation of conversion factors was described in conjunction with Figures 5 and 6 of paper 1. Results in the smallest $\beta_b$ box are unreliable because many points in this outermost region had to be discarded due to low count rates. Since the thickness of each ($x, y, \beta_b$) box is calculated independently, the uncertainty in box thicknesses can be estimated by examining fluctuations between the thicknesses calculated for adjacent boxes. Such fluctuations are typically $\sim 20\%$ or less so we estimate errors in the thicknesses to be 20%. If the CPS is defined as the region with $\beta_b > 1$, Figure 1 shows that the CPS beyond $x = -13$ $R_E$ has a half thickness of 3 $R_E$.
near midnight and 5 $R_E$ in the flanks. These are estimates of the half thicknesses of the region in which about $1/\sqrt{2}$ or 70% of the current flows in the average magnetotail. The techniques used to prepare Figure 1 avoid the artificial inflation of thickness estimates that could be produced by averaging effects of tail flapping and changes in the tilting and warping of the neutral sheet in response to fluctuations in the IMF direction.

2.2. Volume Current Density

[13] Figure 2 shows $j_y(z)$ in nA/m$^2$ calculated separately for each $x$-$y$ box. This plot was created at the fixed set of $z$ locations given in the figure caption by interpolating between the boxes whose edges are given by the information in Figure 1. Paterson et al. [1998] and paper 1 showed that current densities as small as these cannot be measured in individual $6 \times 6$ $R_E$ $x$-$y$ boxes by evaluating the difference between ion and electron drift velocities. Figure 2 was created by calculating $\nabla \times B$ for each box. Combining the 3-D model magnetic field with Ampere's law therefore provides a way to determine average current densities that cannot be measured directly. The difference between ion and electron cross-tail drift velocities required to produce the current densities shown for most boxes in Figure 2 is 5 to 10 km/s. These estimates are based almost entirely upon the relatively accurate measurements of $\mathbf{P}$ and $\mathbf{B}$. The difficult measurements of electron drift velocity are not used at all. The ion velocity measurements contribute only to the small ion inertia term of the momentum equation in the modeling method used to produce Figure 2 and are not used at all in the method described in the appendix.

[14] The set of eight $z$ boxes shown in Figure 2 was used throughout this paper. Although the plot of $j_y$ obtained by evaluating $\nabla \times B$ separately for each box is jagged, the currents are consistently positive and usually tend to decrease as one moves away from the neutral sheet. Paper 1, which was based on sorting by $\beta_x$ rather than $\beta_x$, found current densities in the $\beta_x > 30$ box to be several times larger than those shown in Figure 2. This shows that the largest values of $\beta_x$ are seen only in unusually thin current sheets where $j_y$ must be abnormally strong, and that sorting by $\beta_x$ can be used to study properties of these thin sheets.

[15] The average magnitude of the current density in the $x$ direction was substantially smaller than $j_y$. Both volume current densities $j_x(z)$ and $j_y(z)$ were calculated for each box and were integrated from the neutral sheet to each box edge to get the total sheet current densities $K_x(z)$ and $K_y(z)$ in mA/m. Integration provides a simple way to detect patterns in average fluid parameters that are too jagged to study easily using individual boxes. Figure 3 shows that a clear pattern exists in $K_x(z)$. The average drift of ions with respect to electrons associated with the observed $j_x$ is $\sim 2$ km/s. This average relative drift speed is much smaller than typical instantaneous ion and electron bulk flow velocities.

3. Pressure Anisotropies

3.1. Magnetic Field Line Shape

[16] Figure 4 shows a set of magnetic field lines traced in the model magnetic field. Simple linear interpolation

Figure 1. Calculated $z$ distance from the neutral sheet to each $\beta_x$ box edge for the twenty $x$-$y$ boxes. The eight $\beta_x$ ranges are $\beta_x > 100$, $30 < \beta_x < 100$, $10 < \beta_x < 30$, $3 < \beta_x < 10$, $1 < \beta_x < 3$, $0.3 < \beta_x < 1$, $0.1 < \beta_x < 0.3$, and $\beta_x < 0.1$.

Figure 2. Cross-tail volume current densities $j_y$ within each $z$ box for the 20 $x$-$y$ boxes. Edges 1 to 8 are located at $z = 0.2, 0.7, 1.5, 2.5, 3.5, 4.5, 5.5, \text{and } 6.5 R_E$, respectively.

Figure 3. Similar to Figure 2 except showing $K_x$, the $x$ component of the integrated sheet current density.
between boxes in each of the three directions was used to find $B$ at any desired $(x, y, z)$ [Press et al., 1986a, p. 95] and tracing used the Bulirsch-Stoer method [Press et al., 1986b, p. 563]. This is the same way that the plasma parameters were evaluated to make the 3-D model. The use of the same procedure for both field and fluid parameters makes comparisons more reliable than they would be if the plasma parameters were compared with a standard model magnetic field. A drawback is that the models are available only in the region directly sampled by Geotail. The modeling technique therefore cannot be extended to the ionosphere.

[17] Since the IMF data were not merged with the data set used here, the magnetic field model was compared with the T89 [Tsyganenko, 1989] model, which is parameterized by $K_p$. The plasma sheet thickness to the point at which $B_y = 1$ or $K_y(z) = K_y(z_l)/\sqrt{2}$ was $\sim 20\%$ larger in the model shown in Figure 4 than in the T89 model. In the above, $K_y(z_l)$ is the current density integrated all the way from the neutral sheet to the lobe. Kaymaz et al. [1994] evaluated $r/C^2 B$ using IMP 8 data to estimate $j_y$ at $x = 33 \text{ RE}$. These resulting estimates of current sheet thickness were a little larger than the thicknesses shown in Figure 1.

[18] One feature that is apparent in Figure 4 is that the magnetic field lines near midnight, in the $y = -1.5$ and $y = 1.5 \text{ RE}$ panels, are more sharply bent near the equator than are field lines near the flanks, in the $y = -13.5$ and $y = 13.5 \text{ RE}$ panels. This is because near the neutral sheet $j_y$ is larger at midnight than in the flanks.

3.2. Electron Anisotropy Data

[19] Figure 5 is the long-term averaged electron anisotropy ratio $P_{\parallel}/P_{\perp}$ which was evaluated along the field lines shown in Figure 4. Although the difference between $P_{\parallel}$ and $P_{\perp}$ is small, usually less than 10%, the trends are clear. The anisotropy was plotted as a function of $x$ in Figure 4 to make it easy to see which anisotropy curve corresponds to each field line in Figure 4. A plot of the same electron anisotropy information as a function of $z$ shows $P_{\parallel}/P_{\perp}$ increasing fairly rapidly between $z = 0$ and $z = 1 \text{ RE}$. The anisotropy ratio exhibits a broad peak at $z = 2$ to $2.5 \text{ RE}$ at midnight and at $z = 3$ to $4 \text{ RE}$ in the flanks.

[20] The electrons have such small masses and energies that they follow spiral guiding center orbits even at the neutral sheet. The significance of Figure 5 is that although the average $P_{\parallel}/P_{\perp}$ is near 1.00 at the equator on all field lines shown, this ratio increases as one moves away from the equator along each field line. If only mirror forces acted on electrons, an isotropic distribution near the equator would remain isotropic along the rest of the field line. Since Figure 5 clearly shows that $P_{\parallel}$ becomes larger than $P_{\perp}$, there must be another force acting on the electrons. The presence of a parallel electric field $E_{\parallel}$ is the most obvious cause of this force. The physical source of such a field near the neutral sheet was evident in orbit tracing studies [Kaufmann and Lu, 1993]. We had to include an $E_z$, which has a large parallel component near $z = 0$, in order to maintain charge neutrality in a region containing guiding center electron orbits and chaotic or nonguiding center ion orbits. Pritchett and Coroniti [1995] also found that an $E_z$ arose in kinetic simulations of a thin current sheet.

[21] Several factors should be considered before drawing physical conclusions from an analysis of the data in Figure 5.

Figure 4. The $x, z$ projections of eight magnetic field lines are shown in each panel. The lines start at $z = 0$ and at the $y$ locations listed at the right side of the figure.
First, the standard deviation of the 1-min averaged data points that contributed to each \((x, y, \beta_s)\) box was \(\sim 0.1\). The standard errors of the averages plotted in Figure 5 should equal this standard deviation divided by \(\sqrt{N}\), where \(N\) is the number of independent data measurements in each box. A typical \((x, y, \beta_s)\) box contains 1500 data points. However, the points taken during one orbit are not independent. Variations of fluid parameters such as \(P_\parallel/P_\perp\) tend to be smaller for a sequence of points while the satellite remains in an \((x, y, \beta_s)\) box than are the orbit-to-orbit variations. A typical \((x, y, \beta_s)\) box contains measurements from fewer than 50 orbits, so \(N\) should be somewhere between \(\sim 50\) and \(\sim 1500\) for the data set used here. The box-to-box fluctuations of the \((x, y, \beta_s)\) box averages used to create Figure 5 are \(\sim 0.01\), suggesting that using \(N\) of \(\sim 100\) gives a reasonable estimate of the standard error.

Other factors to consider involve the use of averaged fluid parameters and average field lines. With regard to the use of average field lines it is evident from Figure 5 that the anisotropy increases from \(\sim 1.00\) to \(1.10\) as one moves from the neutral sheet to the middle CPS along essentially any path. Errors in the shapes of field lines will therefore have little effect on the conclusion that \(P_\parallel/P_\perp\) increases by \(10\%\) as one follows a field line away from the neutral sheet.

We also checked to see if the average anisotropies were produced by combining a small number of large \(P_\parallel/P_\perp\) points with many points having \(P_\parallel/P_\perp\) near 1.0. This was not the case. There were roughly equal numbers of points with \(P_\parallel/P_\perp < 1\) and with \(P_\parallel/P_\perp > 1\) very near the neutral sheet. However, only \(5\%\) of the points had anisotropies less than \(0.98\)–\(1.00\) and \(5\%\) had anisotropies greater than \(1.2\)–\(1.3\) in each box in the middle and outer CPS. Only \(\sim 0.1\%\) of the points had anisotropy ratios exceeding \(2\). The bulk of the data points therefore had anisotropies near 1.1 in the middle and outer CPS. These results show that the features in Figure 5 were not produced by a few data points with very large anisotropies but that the features are typical of the plasma sheet.

### 3.3. Analysis of Electron Anisotropies

The following analysis relates the electric potential along any one field line \(\phi(s)\) to changes in the electron anisotropy on that field line. The variable \(s\) is arc length from the neutral sheet and the potential is picked so that \(\phi(0) = 0\). If the electric field always pointed away from the neutral sheet we would have \(\phi(s) \leq 0\) everywhere and electrons moving away from the neutral sheet would be decelerated. If \(f(\mathbf{v})\) is a biMaxwellian distribution function at the neutral sheet, then \(f(\mathbf{v})\) at a point with a given \(\phi(s) \leq 0\) is

\[
\begin{align*}
\frac{f(\mathbf{v})}{n_0} & = n_0 \exp \left[ \frac{e\phi}{T_{\parallel i0}} - \frac{m}{2\pi T_{\parallel i0}} \right]^{3/2} \exp \left[ -\frac{m(v_x^2 + v_y^2)}{2T_{\parallel i0}} \right] \\
\frac{r(s)}{1 + (R - 1) \frac{B(\mathbf{s})}{B_0}} & \\
& = 1 + (R - 1) \frac{B(\mathbf{s})}{B_0},
\end{align*}
\]

where \(R = T_{\parallel i0}/T_{\parallel i0}\) and \(n_0, B_0, m,\) and \(e\) are the density and magnetic field strength at the neutral sheet, the electron mass and the magnitude of the electron charge. The parameters \(T_{\parallel i0}\) and \(T_{\parallel i0}\) are the parallel and perpendicular thermal energies at the neutral sheet. These two parameters and \(R\) are fixed along any one field line. The thermal energies and pressures vary along a field line, and are given by \(P_{\parallel i}(s) = n(s)T_{\parallel i0}\), \(P_{\parallel i}(s) = n(s)T_{\parallel i0}/r(s)\). The density \(n(s)\) varies according to

\[
\frac{n(s)}{r(s)} \exp \left[ \frac{\phi(s)}{T_{\parallel i0}} \right] = \frac{n_0 R}{n_0 R} \exp \left[ \frac{\phi(s)}{T_{\parallel i0}} \right]
\]
If $T_{\|0} = T_{\perp0}$ so that $f(v)$ is isotropic at the neutral sheet then $R = 1$, giving $r(s) = 1$. This particular $f(v)$ therefore remains isotropic everywhere along a field line on which $\phi(s) < 0$.

[25] Figure 6a is a sketch of (1) with $R = 1$. Although the flux levels depend on location $s$, the basic structure of this $f(v)$ remains unchanged all along a field line on which $\phi(s) \leq 0$. All sketches in Figure 6 were prepared for an $f(v)$ that is isotropic at the neutral sheet, though equations (1) and (2) are valid for any bi-Maxwellian.

[26] Strong anisotropies can develop in a distribution that is isotropic at the neutral sheet if $E_k$ points toward the neutral sheet so that $\phi(s) > 0$. The orbit tracing studies presented by Larson and Kaufmann [1996] showed that the density can increase or decrease as one moves along a field line away from the neutral sheet. Decreases are expected if $\phi(s) < 0$ and increases if $\phi(s) > 0$. In this latter case electrons are accelerated by $E_\|\$ through a potential drop $f(v)$ as they move away from the neutral sheet. If no instabilities developed $f(v)$ would split into sets of two hemispherical contours, one going away from the neutral sheet and one returning after having mirrored at lower altitudes (Figure 6b). The hemispherical shells would have low energy cutoffs so there would be no electrons with parallel velocities less than $v_c(s) = \frac{2e\phi(s)}{m}$. Since these sharp edges in $f(v)$ would be highly unstable, it was assumed that electrons will diffuse in the parallel direction as a result of Landau interactions. The final result would be cylindrically shaped contours of constant $f(v)$ at $|v_\perp| < v_c$ and spherical end caps starting at $\pm v_c$ (Figure 6c). This $f(v)$ is similar to electron distribution functions sometimes observed in Geotail data [Paterson et al., 1998].

[27] The distribution in Figure 6c could remain stable once it formed so strong waves and diffusion would not necessarily be seen at the location of such an $f(v)$. Particles in the hemispherical end caps would have less kinetic energy at locations closer to the neutral sheet. At such locations the group of electrons moving away from $z = 0$ would not have been fully accelerated and those moving toward $z = 0$ after having mirrored would have lost some of their peak kinetic energy. All electrons in the cylindrical portion of $f(v)$ are electrostatically reflected before reaching $z = 0$, so $v_c$ decreases and the two end caps simply merge at the neutral sheet to produce an $f(v)$ similar to that shown if Figure 6a. The distribution functions sketched in Figure 6c are [Lu, 1993]

$$f(v) = n_0 \exp \left[ \frac{e\phi}{T} \right] \frac{m}{2\pi T} \frac{3/2}{\exp \left[ \frac{m(v_\perp^2 + v_\|^2)}{2T} \right]} |v_\| > v_c$$

$$f(v) = n_0 \left[ \frac{m}{2\pi T} \right]^{3/2} \exp \left[ -\frac{mv_\|^2}{2T} \right] |v_\| < v_c$$

where $T_{\|0} = T_{\perp0} = T$ has been assumed and $\phi(s) \geq 0$. Integrating equation (3) yields a density

$$\frac{n}{n_0} = 2 \left[ \frac{e\phi}{\pi T} \right]^{1/2} + \left[ 1 - \text{erf} \sqrt{\frac{e\phi}{T}} \right] \exp \left[ \frac{e\phi}{T} \right]$$
and a $P_\parallel/P_\perp$ ratio given by $P_\perp = nT$ and

$$P_\parallel = nT \left[ 1 + \frac{4 \lambda^3}{3\sqrt{\pi}[1 - \text{erf}(\lambda)]} \exp(\lambda^2) + 6\lambda \right]$$ (5)

where $\lambda^2 = e\phi / T$.

[28] Figure 7 is a plot relating $P_\parallel/P_\perp$ to $\lambda$ as given by equation (5). For example, $P_\parallel/P_\perp = 1.10$ corresponds to $\lambda = 0.54$. Since the average electron thermal energy beyond $x = -20 R_E$ is 600 eV, an average maximum $P_\parallel/P_\perp = 1.10$ implies a total potential drop of $\phi = 180$ V between the neutral sheet and the middle CPS. Using 2 $R_E$ as the $z$ distance over which this potential drop exists yields an average electric field $E_z = 14$ $\mu$V/m. This is much smaller than the electric fields that can be directly measured by electric field detectors. Although small, this parallel electric field is important because it maintains charge neutrality, produces either electron beams or stretched electron distribution functions, and because it violates the ideal MHD assumptions.

### 3.4. Ion Anisotropy Data and Analysis

[29] Figure 8 shows the ion anisotropy in the same format that was used for electrons in Figure 5. Kistler et al. [1992], using data from the AMPTE CCE and IRM satellites, also noted that the ion pressure ratio is usually $1.05 \pm 0.05$. Although the magnitudes of the ion and electron anisotropies are similar, the causes are different. The ions are at least 5 times more energetic than the electrons beyond $x = -15 R_E$ [Baumjohann et al., 1989]. Equation (5) shows that the same $|E|_{\parallel}$ that caused the electron $P_\parallel$ to be 10% larger than $P_\perp$ would cause the more energetic ions to have $P_\parallel$ less than 1% larger than the ion $P_\perp$.

[30] The shapes of the ion anisotropy curves can be understood as a consequence of mirror forces in the region of ion guiding center motion. When plotted as a function of $z$, the data in Figure 8 show that the ion anisotropy on field lines that intersect the neutral sheet between $x = -30$ and $x = -20 R_E$ peaks slightly below $z = 3 R_E$ at midnight and near 4 $R_E$ in the flanks. The anisotropy on these field lines nearly disappears at $z$ between 4 and 5 $R_E$ at midnight and at $z = 4.5$ to 5.5 $R_E$ in the flanks. Deviations from isotropy are

![Figure 7. Anisotropy ratios calculated using the model sketched in Figure 6c.](image)

![Figure 8. Similar to Figure 5 except for ions.](image)
smaller on field lines that intersect the neutral sheet earthward of $x = -20 R_E$.

[11] A simple explanation of these observations is based on the assumption that the net effect of chaotic ion motion at $z < 1 R_E$ is to generate an $f(v)$ that is composed of an isotropic component plus a source cone component with a sharp cutoff at pitch angle $\alpha$. As with electrons, nearly all downgoing source cone particles would mirror and return to the neutral sheet producing a bidirectional $f(v)$. The very small fraction of ions that actually reach the ionosphere and are lost cover such a small pitch angle range that their absence could not be observed. Moving down the field lines from $z = 1 R_E$, which is near the point at which ion orbits begin to follow spiral guiding center trajectories, the mirror force would increase all ion pitch angles and therefore the cutoff angle $\alpha$. If isolated, the $P_\parallel/P_\perp$ ratio of this source cone distribution would be largest at $z = 1 R_E$, the beginning of the guiding center region. However, these particles cover such a small solid angle range that they contribute little to the anisotropy of the total distribution composed of source cone plus isotropic components. Moving away from $z = 1 R_E$ the increase in the anisotropy of the combined $f(v)$ is produced by the increasing importance of source cone particles as $\alpha$ increases and they fill a larger region of velocity space. In this region the increase in the importance of source cone particles due to the increasing phase space occupied dominates over the decrease in the initially very large $P_\parallel/P_\perp$ ratio of this component. Continuing down the field line the source cone particles will eventually extend over all pitch angles and therefore become isotropic. As a result, the overall anisotropy peaks and then decreases until $P_\parallel/P_\perp$ approaches unity.

[32] The model $f(v)$ used to analyze Figure 8 consists of an isotropic portion with density $n_0$ and thermal energy $T_0$ plus a bidirectional source cone component with thermal energy $T_1$. The relative magnitudes of the source cone to isotropic contributions to $f(v)$ is $n_1 = n_1/n_0$. The combined $f(v)$ has a pressure anisotropy given by

$$\frac{P_\parallel}{P_\perp} = \frac{1 + n_1 T_1 (1 - \cos^3 \alpha)}{1 + n_1 T_1 (1 - 1.5 \cos \alpha + 0.5 \cos^3 \alpha)}$$  \hspace{1cm} (6)$$

where $T_r = T_1/T_0$. Since ions follow spiral guiding center trajectories beyond $z = 1 R_E$ each ion’s magnetic moment $\mu$ and therefore $(1/B) \sin \alpha$ should remain constant along a field line. With $T_1 = 1$ the function $P_\parallel/P_\perp$ in equation (6) peaks at the typically observed value of 1.10 between $\alpha = 50^\circ$ and $55^\circ$ with $n_1$ between 1/5 and 1/6. The source cone component becomes isotropic when the cutoff angle reaches $\alpha = 90^\circ$ or when $\sin^2 \alpha = B_p/B_r$, where $\alpha_p$ and $B_p$ refer to values at the point where $P_\parallel/P_\perp$ peaks and $B_r$ to the point at which $f(v)$ becomes isotropic.

[13] Combining the observations in Figure 8 with a plot of $B$ along each field line showed that $B_p$ was between 19 and 24 nT at midnight and $\sim 19$ nT in the flanks for all field lines which intersect the neutral sheet between $x = -30$ and $-20 R_E$. The anisotropy ratios in Figure 8 all flatten out near the Earth at $P_\parallel/P_\perp \sim 1.02$. Using this point as the point at which $f(v)$ becomes nearly isotropic gives $B_p$ between 35 and 41 nT at midnight and between 31 and 37 nT in the flanks. Using only these observed field strengths and the constancy of $\mu$ gives $\alpha_p$ between 43$^\circ$ and 56$^\circ$. This result agrees reasonably well with the prediction of 50$^\circ$ to 55$^\circ$ from (6). Picking $\alpha_p = 50^\circ$ and again assuming $\mu$ is constant beyond $z = 1 R_E$ gives $\alpha_p$ between $22^\circ$ and $28^\circ$, where $\alpha_p$ is the source cone width at $z = 1 R_E$, the edge of the chaotic region.

[34] The number density of the isotropic component is $n_0$ everywhere along a field line. The bidirectional source cone component adds an additional $n(s) = n_1 [1 - \cos (s)]$, where $\alpha(s)$ is the cutoff pitch angle at the point $s$. The values of $n_1$ and $\alpha$ used above suggest that the ion density should increase by 15 to 20% as one moves down an average field line from $|z| = 1 R_E$ to the point at which the anisotropy nearly disappears. Plots of $n_1$ and of $n_1$ along field lines at $|z| > 10 R_E$ do show increases of this magnitude. However, similar plots on field lines closer to the flanks show more complicated variations. Although 10% variations of ratios such as $P_\parallel/P_\perp$ are reliably measured, individual parameters have larger box-to-box fluctuations.

[35] To summarize the above results, the observed ion anisotropies on field lines reaching the neutral sheet between $x = -30$ and $-20 R_E$ can be explained if the net effect of chaotic ion motion near the neutral sheet is to produce a two-component $f(v)$ at $|z| = 1 R_E$. The principal component is an isotropic Maxwellian. A smaller source cone component has a cutoff near $\alpha = 25^\circ$. The source cone distribution becomes bidirectional because nearly all outgoing ions mirror and return to the neutral sheet.

Equation (6) shows that the source cone contribution makes less than a 20% contribution ($n_1 \leq 1/5$) to the total $f(v)$. This minor deviation from isotropy would be difficult to detect in individual 1-min averaged $f(v)$ plots. Beyond $|z| = 1 R_E$ the ion magnetic moments are conserved. Equation (6) shows how the mirror effect causes both the increase of $P_\parallel/P_\perp$ to a peak near 1.10 and the subsequent decrease of $P_\parallel/P_\perp$ to near unity as one moves down a field line. A small increase in density is also predicted but would be difficult to measure in the presence of large fluctuations.

4. Summary

[36] This paper presented the first results from an analysis of 3-D database models created using observations from the Geotail CPI and MGF detectors. A simple method to convert from $(x, y, z)$ to $(x, y, z)$ coordinates was described in paper 1. The appendix outlines a more comprehensive procedure, which was used here primarily to verify the accuracy of the simpler method. The complex procedure in the appendix produced stable results only when many years of data were combined using the large $6 \times 6 R_E$ $x-y$ boxes. Fluctuations created instabilities when using this new procedure with less data or smaller box sizes.

[37] The 3-D aspect of the models is needed to carry out the studies described here and for many other possible related studies. With 3-D models one can evaluate all derivatives that are needed to study the structure of the plasma sheet. It also is possible to trace field lines and to study changes of fluid parameters seen as one moves along these average field lines. Studies of this type cannot be carried out with 2-D models such as those obtained by averaging all data with $\beta$ greater than a fixed cutoff.
A common property of the analyses described in this paper is that they were able to determine values of parameters that were too small to be measured directly. The first parameters studied were $j_x$ and $j_y$. These were derived by taking the curl of the model $B$. This method provided reliable current estimates throughout the region studied. Both averages in individual boxes, as illustrated for $j_x$ in Figure 2, and averages from the neutral sheet to each box edge, as illustrated for $j_y$ in Figure 3, were carried out for all plasma and field parameters in the 3-D models. The average $j_y$ was $\sim 0.5$ nA/m², and $|j_x|$ varied from $<0.2$ nA/m² everywhere beyond $x = -20 R_E$ to as much as $0.5$ nA/m² at some points near $x = -10 R_E$. In paper 1 it was seen that direct measurements of $j_y$ using particle detectors have large uncertainties even when $x-y$ box sizes were increased to $22 \times 6 R_E$. Paper 1 showed that direct measurements were unable to determine $j_x$ even with this large box size.

The physics of the current patterns shown in Figures 2 and 3 is easy to understand. The $j \times B$ force associated with $j_y$ has an earthward component that balances the tailward pressure gradient force and therefore keeps the higher-pressure plasma near the Earth from expanding into the tail. Both $x$ and $y$ currents produce a force toward the neutral sheet, keeping the plasma sheet confined in the $z$ direction. The $x$ current also produces a $j \times B$ force toward midnight from each flank, allowing higher-pressure plasma to be confined near midnight. In terms of associated magnetic fields, $j_x$ creates the principal $B_x$ field in the lobes while the $x$ currents shown in Figure 3 produce the $B_y$ associated with flaring of the tail as one moves away from the Earth.

The other 3-D model parameters that were analyzed are the electron and ion pressure anisotropies $P_x/P_z$. Since the ordinary mirror effect changes $v_i$ and $v_x$ of each particle as it moves along a field line, the information needed for an analysis is the change in the anisotropy ratio that is seen as one moves along a field line. Average field lines therefore were first traced throughout the region studied. The interpolation procedures used to determine points along the field lines were also used to determine the anisotropy ratios at these same points. This makes use of the 3-D model more internally consistent than an evaluation of the ratios along field lines from any standard magnetosphere model would be.

The anisotropy ratios for ions and electrons both were close to unity at the neutral sheet. The ion and electron ratios varied in different manners along the field lines, but each reached a maximum near 1.10 within 2 to 4 $R_E$ of the neutral sheet and then decreased at lower altitudes. Although these variations showed qualitative similarities, it was concluded that they were caused by different effects.

Changes in the electron anisotropy were attributed to the presence of a weak parallel electric field $E_{||}$ near the neutral sheet. Previous orbit tracing studies and simulations showed that such a field is needed to maintain charge neutrality in a region containing guiding center electron and nonguiding center ion orbits. The average $E_{||}$ needed within $2 R_E$ of the neutral sheet was estimated to be $14 \pm 1 \mu$V/m, which is smaller than the electric fields that can be measured directly.

Although small, the presence of an $E_{||}$ is significant because it violates the ideal MHD assumptions. The total potential drop of 200 V is much smaller than the several kV potential drops seen at low altitudes on auroral field lines. However, the magnetic field strength near the neutral sheet also is much smaller than fields near the Earth. Since $E_{||}$ was attributed to the need to maintain charge neutrality it is evident that the parallel electric force $eE_{||}$ must be comparable to the average mirror force $\mu_0 B \partial \phi / \partial s$ on electrons, where $\mu$ is the magnetic moment. With $E_{||} = 14 \mu$V/m the average electric force is $1/3$ the average mirror force on electrons near the neutral sheet. As was noted by Alfvén and Fälthammar [1963], this implies that particles cannot be assumed to be tied to magnetic field lines near the neutral sheet. This breakdown of the ideal MHD assumptions could be particularly significant for studies involving magnetic reconnection and the growth of some instabilities in the tail.

The same $E_{||}$ that produces 10% changes in the electron anisotropy ratio would create less than a 1% change in the anisotropy of the more energetic ions. The ion anisotropies were explained by assuming that the net effect of nonguiding center ion motion near the neutral sheet is to produce a weak source cone component that is superimposed upon an otherwise isotropic ion distribution function $f(v)$. Ions move in helical guiding center orbits when they are more than $\sim 1 R_E$ from the neutral sheet. It was concluded that the presence of a source cone component that contributes 20% to the total $f(v)$ within the source cone is sufficient to explain all changes in the ion anisotropy beyond $|z| = 1 R_E$. The ordinary mirror effect broadens this source cone cutoff angle from 25° at $|z| = 1 R_E$ to 50° at the point where the anisotropy peaks. Farther down the field line the cutoff angle continues to increase until it reaches 90° so that $f(v)$ again becomes nearly isotropic. The simple source cone model used has $f(v)$ constant out to a sharp cutoff angle. This would result in a perfectly isotropic $f(v)$ at low altitudes. The actual anisotropy ratio remained slightly above 1.00, indicating that the model is oversimplified. A source cone distribution that decreases more smoothly with angle can explain this behavior.

5. Appendix

Paper 1 described the procedure that was used to produce Figure 1, and therefore upon which all other results in the present paper are based. It also was noted in paper 1 that a more comprehensive method to calculate box thicknesses had been developed but that this more complex method suffers from numerical instability. Since determining the thickness of each $\beta_x$ box is the critical step that determines the accuracy of the 3-D models, this appendix will describe the more complex method, summarize the problems found when using each method, and compare results from the two techniques.

The goal is to convert from nonorthogonal $(x, y, \beta_x)$ coordinates to orthogonal $(x, y, z)$ coordinates by using the momentum equation to calculate the thickness of each box. Figure 9a is a 2-D sketch that illustrates the problem with the method used here and in paper 1. The technique determines only a single average thickness for each box. In some respects the final boxes are therefore considered to be rectangular, as shown in Figure 9a. This assumption is clearly incorrect since each box contains all data within a specified $\beta_x$ range. The point at which $\beta_x = 100$, for example, cannot jump to a different $z$ as one moves a small
top will not in general form a plane. We use \( \mathbf{d}_1 \times \mathbf{d}_2 \) to define the normal vector for the box top, where \( \mathbf{d}_1 \) is a vector from one corner of the top to the opposite corner and \( \mathbf{d}_2 \) connects the other two corners of the box top. With this technique the \( x \) pressure force on a box of interest depends partly upon \( P_x \) in the boxes above and below and on the slopes of the top and bottom rather than only on the small element \( P_x \) in these two adjacent boxes.

[48] As in the method described in paper 1, both normal and tangential forces are calculated for each side of a box of interest. To simplify the procedure, the inertial term \( \rho \mathbf{v} \cdot \nabla \mathbf{v} \) was neglected in the distorted box method. The inertial term was included in the rectangular box method, but regularly turned out to be smaller than the box-to-box fluctuations of other terms. Neglecting the inertial term reduces the momentum equation to

\[
\nabla \cdot \mathbf{R} = 0
\]

where \( \mathbf{P}_i \) and \( \mathbf{P}_e \) are the ion and electron pressure tensors, \( \mathbf{I} \) is the unit tensor, and \( \mathbf{BB} \) is a dyad.

[46] The \( x \) component of the force balance expression \( \nabla \cdot \mathbf{R} = 0 \) provides one equation for each box. As described in paper 1, the \( z \) component of the momentum equation was used primarily to check detector calibration, and the \( y \) component had large relative uncertainties so did not give reproducible results. For each row of boxes, as such the row with \( 30 < \beta_z < 100 \), there are \( (N_x + 1)(N_y + 1) \) unknown box edges and only \( N_x N_y \) boxes or equations, where \( N_x \) is the number of boxes in the \( x \) direction. Figure 9c shows the box edges for \( 6 \times 6 R_E \) boxes with \( N_x = 4 \) and \( N_y = 5 \). It was necessary to make \( N_x + N_y + 1 \) assumptions to solve the set of \( N_x N_y \) equations for all remaining box edges. The first assumption made involved symmetry at midnight. Each pair of box edges marked by the squares in Figure 9c, such as the pair at \( (x = -19, y = \pm 3) R_E \), were assumed to have equal \( z \) heights. The other assumption was that the box thicknesses ceased to depend on \( x \) for the box farthest from Earth. The pairs of box edges marked by circles in Figure 9c, such as the pair at \( (x = -31, y = -9) R_E \) and at \( (x = -25, y = -9) R_E \), therefore also were assumed to have equal \( z \) heights. These assumptions made the four edges marked by both circles and squares all equal, so the box with these four edges was rectangular. With \( 6 \times 6 R_E \) boxes these assumptions left \( N_x N_y = 20 \) unknown box edge heights for the 20 boxes in any one row and the 20 associated equations (Figure 9c). Heights of the boxes in each row were calculated separately starting at the neutral sheet, on which all box bottoms are at \( z = 0 \).

[50] Figure 10 compares the results obtained by this more complex calculation with the results from the simple calculations described in paper 1. The format is similar to Figure 1 except that Figure 10 shows the thickness of each individual box while Figure 1 was the total distance from the neutral sheet to each box edge. One other difference is that the first box in Figure 10 covers the range \( \beta_z > 300 \) and the last box contains all \( \beta_z < 0.3 \) data. The end boxes in Figure 1 had \( \beta_z > 100 \) and \( \beta_z < 0.1 \). This use of one more large \( \beta_z \) box and
the distorted box method became erratic when farther from the neutral sheet.

eters cause these heights to oscillate in the layers that are

tum equation, but fluctuations in the observed fluid param-

eters were larger in these runs with less data per box.

This increase in the fluctuation level made the box edge height calculations unstable even for the boxes closest to the neutral sheet.

It is because of this lack of stability that results from the distorted box method were not used in either paper 1 or the present work. It is possible that this complex but physically more satisfying method can be used when more years of data become available and when the $6 \times R_E$ box sizes are appropriate. For now, the distorted box method is used primarily to check the height calculations in boxes for which both methods are stable.

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