Birkeland currents in the plasma sheet

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[1] Geotail particle and magnetic field measurements were combined to generate long-term-averaged 3-D models of the plasma sheet. Ampere’s law was used to calculate the Birkeland current $j_\parallel$ in the $-30 < x < -16R_E$, $|z| > 1R_E$ region. Current diversion, or the growth of current in a unit flux tube $j_\parallel/B$, took place throughout the region studied. This suggests that electron scattering is broadly distributed. No substantial change in $j_\parallel/B$ could be detected between the plasma sheet boundary layer and the ionosphere. Birkeland currents were strongest and exhibited a dawn-dusk asymmetry when the interplanetary magnetic field (IMF) was southward. This asymmetry may be associated with the formation of thin current sheets on the dusk side during disturbed periods. Symmetries were apparent above and below the neutral sheet when the IMF was northward or southward, but these symmetries were not present when the IMF pointed dawnward or duskward. For these latter cases, separate surfaces were found on which $B_x = 0$, $B_z = 0$, and $j_\parallel = 0$. This apparently complex structure could be understood as a consequence of the tendency for $B_x$ in the neutral sheet to have the same sign as the IMF $B_z$. The observed Birkeland currents were in the region 1 sense when leaving the plasma sheet for all IMF orientations. Current diversion was analyzed in an MHD framework. The analysis suggested that the reduction of gradient and curvature guiding center drifts, and the presence of polarization currents in the diversion region can provide sources of electrons to sustain a steady $j_\parallel$. It also was noted that the formation of an $E_y$ region in the topside ionosphere can make it appear to the conducting ionosphere as if it is being driven by a current source rather than by the plasma sheet electric field.

INDEX TERMS: 2764 Magnetospheric Physics: Plasma sheet; 2708 Magnetospheric Physics: Current systems (2409); 2744 Magnetospheric Physics: Magnetotail; 2736 Magnetospheric Physics: Magnetosphere/ionosphere interactions; 2708 Magnetospheric Physics: Magnetotail;

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1. Introduction

[2] Previous studies have determined $\nabla j_\perp$ in the ionosphere and the formation of Birkeland currents $j_\parallel$ at the ionospheric ends of field lines. In this paper we use data from the Geotail satellite to carry out a 3-D analysis of currents within the plasma sheet. The goal is to examine the formation of $j_\parallel$ at the high-altitude ends of nightside auroral zone field lines.

[3] The present work concentrates on the $-30 < x < -8$, $|y| < 15R_E$ portion of the plasma sheet. In the T89 model [Tsyganenko, 1989] this region maps to the nightside ionosphere at magnetic latitudes between $62^\circ$ and $75^\circ$ and to local times within 4 hours LT of midnight [Kaufmann et al., 1993].

[4] Early low-altitude studies of the region 1 and region 2 Birkeland current systems were made by Zmuda and Armstrong [1974] and by Iijima and Potemra [1976a]. More poleward systems, referred to as region 0 currents, were studied starting with the dayside system [Iijima and Potemra, 1976b]. Weimer [2001] created long-term-averaged data-based models of $j_\parallel$ in the ionosphere throughout the auroral zone and polar cap. He also showed how the interplanetary magnetic field (IMF) controls $j_\parallel$. Our results are compared with Weimer’s [2001] model in section 4.

[5] Our reliance on data from a single satellite restricts this work to a study of large-scale long-term-averaged $j_\parallel$ patterns. Most fluid parameters are highly structured on plasma sheet field lines. Peria et al. [2000] noted that an average of 10 current sheets are crossed per pass over the auroral zone and polar cap. Pairs of field-aligned currents often have been seen near auroral arcs. However, it is the patterns seen regularly in the long-term averages of these...
fluctuating and confined structures that were used in the Birkeland current studies noted above. Even though the east-west magnetic field showed many localized perturbations associated with individual arcs and adjacent return currents, there was usually a clear region of increasing and a region of decreasing azimuthal magnetic field which were associated with the broad Birkeland current systems. The wave-dominated region poleward of the relatively steady arcs is likely to connect to the plasma sheet boundary layer (PSBL). Although this is an important part of the auroral zone, it cannot be studied here because particle count rates near the PSBL were too low.

The most comprehensive direct analysis of long-term-averaged parallel currents in the central plasma sheet (CPS) is that of Tsyganenko et al. [1993]. The total parallel current \( J_\parallel(z_{PSBL}) \),

\[
J_\parallel(z) = \int_0^z j_\parallel(z')dz',
\]

integrated from the neutral sheet to the PSBL was evaluated as a function of \( x \) and \( y \). The total region 1 current agreed with observations at the ionospheric ends of the corresponding field lines. Birkeland currents in the magnetotail were found to be larger when the IMF pointed southward than when it was northward. The integrated parallel currents \( J_\parallel \) had an antisymmetric \( z \) dependence with peaks near \( |y| = 8R_E \). Integrated currents decreased at large \( |x| \) because the total current from the neutral sheet to the PSBL involved integration over more field lines at small \( |x| \). The observed \( z \) dependence was strong, with little current originating beyond \( 25R_E \).

Israelevich et al. [2001] used Geotail data to deduce 3-D currents throughout the plasma sheet. Plots of the Cartesian components of \( j \) were presented. The parallel component was not separated out, so the dominant perpendicular currents were emphasized. One feature pointed out in this paper was a dawn-dusk asymmetry in plasma sheet currents.

Wing and Newell [2000] studied the 2-D distribution of parallel currents near the equatorial plane by projecting the plasma pressure measured at low altitudes by the DMSP satellites to the equator. A modified T89 magnetic field model was used. The parallel current was estimated using a method based on the continuity and momentum equations. Since ions dominate the plasma pressure, only ion parameters were used.

Most of the results noted above will be apparent in the analysis that follows and are discussed in the following sections. However, the principal emphasis of the present paper is a study of the volume current density \( j_\parallel \) and of its changes as one moves along an average magnetic flux tube. The emphasis is therefore on the 3-D structure of parallel currents in the plasma sheet rather than on integrated or 2-D current distributions.

2. Parallel Volume Current Density Observations

2.1. Techniques

This subsection summarizes the methods used to create 3-D data-based models of many magnetotail plasma and field parameters. Estimates of uncertainties in \( j_\parallel \) and additional details are contained in Appendix A.

Four years of Geotail satellite observations were used to generate the models. Magnetic field data are from the MGF detectors [Kokubun et al., 1994] and particle data are from the CPI detectors [Frank et al., 1994]. Approximately, 100,000 one minute averaged measurements of each plasma and field parameter were made within the plasma sheet per year. Distribution functions from the separate ion and electron detectors were transmitted to the ground with a 20-s energy and angle scan rate, and then averaged over 1 min to accumulate sufficient counts. All plasma parameters were evaluated on the ground by integration over the ion and electron distribution functions after incorporating background and satellite potential corrections. The lowest ion and electron energies that could be included were dependent on the satellite potential and the count rates. The highest energy bands were centered at 48 keV for both species, and \( \Delta E/E \) was about 10\%. Many more details are given by Frank et al. [1994].

The observations first were sorted into \( 6 \times 6R_E(x, y) \) boxes, where \( x \) and \( y \) are aberrated GSM components of the satellite location. The trajectory was first evaluated in GSM coordinates and then an average aberration angle correction was applied to all data. Tsyganenko et al. [1998] pointed out that a more complex point-by-point aberration correction should be carried out. This difference is not as important here as in most studies because we do not use the \( z \) component of the satellite trajectory anywhere in the analysis. Instead of using trajectory \( z \) information, data were sorted by \( \beta_z \) within each \( (x, y) \) box

\[
\beta_z = 2\nu_0P/B_z^2,
\]

where \( P \) is the isotropic part of the ion plus electron pressure tensor. The parameter \( \beta_z \) is infinite at the neutral sheet, which in this paper is defined as the sheet on which \( B_z = 0 \). The \( z \) thickness of each \( B_z \) box is calculated using the momentum equation in a form appropriate for long-term-averaged models [Kaufmann et al., 2001]

\[
p_r(v_r \cdot \nabla)v_r = -\nabla \cdot P - \nabla \cdot P_e + j \times B,
\]

where \( \rho_r \) is the mass density and \( P_\phi \) is the pressure tensor of species \( \sigma \). In equation (3) it has been assumed that \( \partial/\partial t = 0 \), that the plasma is electrically neutral, and that the force of gravity and electron inertia are negligible. The forces associated with fluctuating electric and magnetic fields are not included in equation (3) because their long-term averages are not available.

The \( z \) component of equation (3) was primarily used to check the absolute calibration of particle detectors. The \( x \) component of equation (3) was used to calculate the \( z \) thickness of each \( \beta_z \) box. All terms in equation (3) were used in the actual thickness calculations. However, for a simple description of the methods only the dominant contributions, which come from the particle and magnetic field pressure gradients and from the magnetic field line tension, will be considered. The \( \beta_z \) sorting ranges were selected so that the \( z \) thickness of each box was much smaller than the \( 6R_E \) dimensions in the \( x \) and \( y \) directions.
The particle and field pressures exert forces in the $x$ direction on the relatively small ends of a box that are nearest to and farthest from the Earth. These forces on a box depend linearly on its $z$ thickness. The largest magnetic field tension force in the $x$ direction is exerted on the large top and bottom of each box, so is not dependent on the $z$ thickness. The calculations adjust the thickness, so long-term-averaged earthward and tailward forces on the box balance. This requirement for average $x$ force balance also was used by Rich et al. [1972] to study the thickness of the current carrying region of the plasma sheet. Kaufmann et al. [2002] give more details.

The resulting $z$ locations provide the information needed to convert from the nonorthogonal ($x, y, \beta_x$) system to the nearly orthogonal ($x, y, z$) system. The ($x, y, z$) system used here is not truly orthogonal because $x$ and $y$ are in aberrated GSM coordinates and $z$ is the distance normal to the average neutral sheet. The neutral sheet is warped and twisted due to seasonal effects and the orientation of the IMF [Kaymaz et al., 1994; Borovsky et al., 1998; Nishida et al., 1998; Tsyganenko et al., 1998]. However, typical $x$ and $y$ derivatives of most plasma and field parameters are only about 10% of the $z$ derivatives. In addition, the difference between $x$ and $y$ derivatives in neutral sheet coordinates and in GSM coordinates is less than 10% as long as the average plasma sheet is distorted by less than 25° from the GSM coordinate system, which is generally true in the region studied here. Errors introduced when taking derivatives using GSM rather than neutral sheet $x$ and $y$ coordinates therefore are small relative to other errors. Since time dependence was neglected in the long-term averages, Ampere’s law reduces to

$$j = \nabla \times \mathbf{B}/\mu_0,$$

$$j_\parallel = \mathbf{B} \cdot \mathbf{j}/\mathbf{B}.$$  

2.2. Magnetic Field Lines

[15] A series of field lines was traced for each set of data selection parameters. Each panel in Figure 1 shows the $x$–$z$ plane projections of a group of eight field lines that started at $z = 0$ and at the $y$ location indicated at the right of the panel. All data taken during the 4 years of observations were combined to create the magnetic field shown in Figure 1. Only the Northern Hemisphere is plotted because Southern Hemisphere data were folded into the Northern Hemisphere by reversing the signs of $B_x$ and $B_y$ whenever the measured $B_x$ was negative. Folding was usually carried out when an examination of separate Northern and Southern Hemisphere data showed clear symmetries. This procedure

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The $x, z$ projections of eight magnetic field lines are shown in each panel. The lines start at $z = 0$ and at the $y$ locations listed at the right side of the panel. Four years of Geotail data were combined to create this model.}
\end{figure}
reduced errors by doubling the number of measurements in each \((x, y, \beta_x)\) box. Details about the use of folded data are in Appendix A.

[16] The method used to determine \(\mathbf{B}\) at any point in the plasma sheet was the same as the method used to determine any of the plasma parameters. Averages were taken throughout each \((x, y, \beta_x)\) box, the box locations were converted to \((x, y, z)\) coordinates, and components of \(\mathbf{B}\) were determined at any desired point by linear interpolation between adjacent boxes [Press et al., 1986a]. This procedure does not impose the restriction that \(\nabla \cdot \mathbf{B} = 0\). Field lines were traced by the Bulirsch-Stoer method [Press et al., 1986b]. Figure 1 shows that field lines near midnight \((y = \pm 1.5R_E)\) panels) are more sharply curved at the neutral sheet than are field lines nearer the flanks \((y = \pm 13.5R_E)\) panels). This is an indication of the concentration of cross-tail current near the neutral sheet at midnight.

### 2.3. Current Density

[17] Figure 2 shows how \(j_{||}\) calculated using equation (5) varies along each of the field lines plotted in Figure 1. The horizontal axis is the \(x\) coordinate in both figures, making it possible to see which field line corresponds to each curve in Figure 2. Since the data were folded, the plots represent currents in the Northern Hemisphere. Positive \(j_{||}\) flows down toward the ionosphere. The \(j_{||}\) in Figure 2 is in the region 1 sense, positive on the dawn side \((y > 0)\) and negative on the dusk side \((y < 0)\). Currents in the Southern Hemisphere also were in the region 1 sense, flowing down to the ionosphere on the dawn side. Figure 2 shows that current densities on the dusk side tend to have larger magnitudes than those on the dawn side. This asymmetry may be associated with the tendency of the current sheet to be thinner on the dusk than on the dawn side (Figure 1).

[18] Two problems are evident in Figure 2. One involves the small abrupt changes in \(j_{||}\) near the neutral sheet ends of most curves. Another feature that appears to be unphysical is that \(j_{||}\) on the more distant field lines tends to peak near \(x = -16R_E\) and then to decrease as one continues earthward. These problems and the tests that were carried out to estimate uncertainties are discussed in Appendix A. The conclusions are that in most cases Birkeland current calculations were reliable only along the \(x \leq -16R_E, |z| > 1R_E\) segment of each field line. It occasionally was possible to obtain \(j_{||}\) beyond this region, as is discussed in later sections.

[19] The parallel current densities are very small. For comparison, average cross-tail current densities are approximately 0.5 nA/m² at \(x = -30R_E\) and 1 nA/m² at \(x = -10R_E\). Although separating the \(x\) and \(y\) contributions in equation (5) has no physical significance, this separation is helpful for parts of the analysis. The \(j_x B_x\) and \(j_y B_y\) contributions to \(j_{||}\) generally have opposite signs when using combined data for all IMF orientations. The magnitude of the \(y\) contribution is

**Figure 2.** Calculated long-term-averaged Birkeland current densities flowing along each of the field lines shown in Figure 1.
usually larger than the magnitude of the \( x \) contribution. It is only the \( y \) contribution to \( j_{IL} \) that is in the region 1 sense. As an example, Figures 3a and 3b show these two contributions which together with \( j_zB_z \) add to produce the total \( j_k \) shown in the \( y = -7.5R_E \) panel of Figure 2. The contribution from \( j_zB_z \) is very small. It may be noted that Figures 3a and 3b are not the \( x \) and \( y \) components of \( j_k \) but are the contributions to \( j_k \) associated with \( j_x \) and \( j_y \).

[20] Since \( B \) is primarily in the \( x \) direction, one might at first guess that it is primarily \( j_x \) that determines \( j_k \) in the outer plasma sheet. Figures 3a and 3b show that not only is this wrong, but that when data for all IMF conditions are combined the net \( j_{IL} \) is in the opposite direction from that given by \( j_zB_z \). The reason the two contributions to equation (5) nearly cancel in Figures 3a and 3b is because the average \( j \) and the average \( B \) tend to be roughly perpendicular. This near cancellation can produce large relative errors. The vectors \( j \) and \( B \) are most nearly orthogonal near midnight, resulting in the smallest \( j_{IL} \) and the largest relative errors. Since absolute errors in \( j_{IL} \) are comparable at all \( y \) locations, the degree to which \( j_{IL} \) approaches zero near midnight in Figure 2 gives a crude estimate of the absolute errors in the calculations. This check shows that statistical errors produced by averaging 4 years of data are about 0.02 nA/m\(^2\). It will be seen later that statistical errors are substantially larger in runs using smaller data sets.

2.4. Currents Within an Average Flux Tube

[21] Plots of \( j_{IL} \) are useful to identify significant features because uncertainties are approximately the same throughout such plots. However, the ratio \( j_{IL}/B \), the parallel current within a unit flux tube, is of more physical interest when

Figure 3. Birkeland currents, contributions to \( j_{IL} \) and currents per unit flux at \( y = 7.5 \) and \( y = -7.5R_E \). These plots are used to examine statistical and systematic errors.
studying current diversion. The volume current density \( j \) will change as one moves along \( B \) if all current carriers remain within a flux tube because \( 1/B \), the cross-sectional area of a unit flux tube, changes. Through the steady state continuity equation \( \nabla \cdot j = 0 \), any change of \( j/B \) along a flux tube indicates the presence of a region in which perpendicular current is locally being diverted to produce parallel current.

[22] A goal of this work is to identify the high-altitude Birkeland current source location by finding where along a flux tube \( j/B \) increases from zero somewhere within the plasma sheet to a maximum near the PSBL. Figure 4 shows \( j/B \) throughout the region in which results were reliable. The Birkeland current in most unit flux tubes built up relatively smoothly throughout the region that could be studied. In particular it is seen that, when observations for all IMF conditions are combined, current diversion is not confined to the \( |z| < 1R_E \) region in which ions follow chaotic orbits nor to the immediate vicinity of the PSBL.

3. Observations Sorted by the IMF Direction

3.1. Northward and Southward IMF

[23] Plasma sheet processes, auroral activity, and Birkeland currents depend on the direction of the IMF. Full analyses, including the derivation of new magnetic field models, therefore were carried out using Geotail data that were sorted according to the IMF clock angle. The method used to transpose solar wind data from the Wind satellite to Earth is described in Appendix A. In order to maintain a reasonable number of data points in each \((x, y, z)\) box the data were sorted into four overlapping \(120/\pi\) sets centered at \(0^\circ\) (IMF \( B_z > 0 \)), \(90^\circ\) (IMF \( B_y > 0 \)), \(180^\circ\) (IMF \( B_z < 0 \)), and \(270^\circ\) (IMF \( B_y < 0 \)). The IMF \( B_x \) and \( B_y \) are strongly correlated because field lines generally follow the Parker spiral orientation. There are not enough data to study independent \( B_x \) effects in fixed \((B_y, B_z)\) ranges. Figures 5 and 6 show plots of \( j/B \) based on northward and southward IMF data, respectively. Each data set showed clear symmetries about \( z = 0 \) so the data again were folded. These figures have different scales, and only results for the \( x/\pi > 16 \) \( R_E \), \( |z| > 1R_E \) portion of each flux tube are plotted. The uncertainties in Figures 5 and 6 are larger than in Figure 4 because each sorted run contained only \( 1/3 \) as much data as was used to prepare the unsorted plots. Even the trends are difficult to see in Figure 5 because the magnitude of \( j \) is so small when the IMF \( B_z > 0 \).

[24] Figure 6 shows that a dawn-dusk asymmetry existed when the IMF was southward. This asymmetry also was seen by Israelevich et al. [2001]. Field line plots similar to Figure 1 showed that the average current sheet was thinnest when the IMF pointed southward. The plasma sheet volume current densities tend to be largest when the current sheet is thinnest because a specific total integrated cross-tail current is required by Ampere’s law to be consistent with a given \( B_z \) in the lobes. Similarly, the integrated \( x \) current is fixed by

![Figure 4](image-url). Current per unit flux tube along each of the field lines shown in Figure 1. Results are plotted only in the \(-30 < x < -16R_E, \ |z| > 1R_E \) segment of each flux tube, where results are reliable.
3.2. Dawnward and Duskward IMF

[25] Figures 5 and 6 for northward and southward IMF were prepared using folded data and showed the qualitative features that were expected. In contrast, \( j_k \) did not exhibit symmetries across the neutral sheet when the IMF pointed primarily dawnward or duskward. It therefore was necessary to keep Northern and Southern Hemisphere data separate for these cases.

[26] Figure 7 shows \( j_k \) on the dawn side \((y < 0)\) when the IMF \( B_y < 0 \) from runs using only plasma sheet \( B_x > 0 \) (Northern Hemisphere) data and from separate runs using only plasma sheet \( B_x < 0 \) (Southern Hemisphere) data. Figure 8 shows the corresponding dusk side \((y > 0)\) results, again based on IMF \( B_y < 0 \) data. The horizontal axis is \( z \) in both these plots. The plots are of \( j_k \) rather than \( j_k/B \), making uncertainties uniform throughout all panels. Since data were sorted using two parameters, the IMF direction and the sign of \( B_x \) within the plasma sheet, each run used only 1/6 as much data as was available to prepare Figure 4.

[27] The behavior of \( j_k \) is clearly different from the simple pattern found in Figures 2 and 4–6. Rather than having the smallest \( j_k \) near midnight, Figures 7 and 8 show some of the largest \( j_k \) at \( y = \pm 1.5R_E \). Rather than having the smallest \( j_k \) near \( z = 0 \), \( j_k \) is sometimes largest near \( z = 0 \). Birkeland currents are seen to be zero at \( z = 2 \) to \( 3R_E \) on the dawn side (Figure 7) and at \( z = -2 \) to \( -3R_E \) on the dusk side (Figure 8).

[28] Figure 9 summarises the rather complex appearing IMF \( B_y < 0 \) results shown in Figures 7 and 8. Figure 9 shows the points at which \( j_k = 0 \) using data from the \( x = -16R_E \) boxes, where the currents are strongest and the calculations are most reliable. The shaded regions are the only flux tubes in which \( j_k > 0 \). Plots of \( B_x \) also were made for the runs associated with Figures 7 and 8. The \( B_x = 0 \) curves in Figure 9 were taken from the \( x = -16R_E \) boxes of these plots. Finally, the heavy lines at the largest \( |z| \) in Figure 9 show the location of the \( B_x = 0.3 \) box edge at \( x = -16R_E \), which represents the outermost point where thickness calculations were reliable. This is near the PSBL. The \( z = 0 \) line is the neutral sheet as defined by the surface on which plasma sheet \( B_x = 0 \). The above observations are discussed in section 5.7.

4. Comparison of Plasma Sheet and Ionospheric Currents

[29] The currents shown in Figures 4–8 can be compared with Birkeland currents measured near the ionosphere. Figure 6 contains the largest currents and therefore the most reliable results. This figure shows that currents at \(|y| < 9R_E\) appear to flatten out at a maximum \(|j_k/B|\) before reaching the \( x = -16R_E \) limit of this study. In contrast, \(|j_k/B|\) is still increasing at \( x = -16R_E \) for the \(|y| > 9R_E \) field lines. This suggests that we have determined the full current in a flux tube only on field lines at \(|y| < 9R_E \). This is the only region for which quantitative comparisons to ionospheric models are reasonable. Runs using \( 3 \times 3R_E \) \((x,y)\) boxes produced useful results on some field lines as far earthward as
\(x = -11.5R_E\) when data for all IMF directions were combined. The \(3 \times 3R_E\) plots showed \(|j_f/B|\) increasing to what appears to be a constant level of about 10 mA/Wb near \(x = -11.5R_E\) for \(|y| = 10.5R_E\). The value of \(|j_f/B|\) still appeared to be increasing at \(x = -11.5R_E\) on the \(|y| = 13.5R_E\) field lines. The value of \(|j_f/B|\) is still increasing at \(x = -11.5R_E\) on the \(|y| = 13.5R_E\) field lines.

[30] A current per unit flux tube of 20 mA/Wb corresponds to \(j_f\) of 1 \(\mu A/m^2\) in a typical auroral ionospheric region where \(B = 0.5\) G. Figures 5 and 6 therefore suggest that the maximum average Birkeland current density at the ionospheric location that at a given instant is connected to the field lines studied here should be 0.7–1 \(\mu A/m^2\) when the IMF points southward (\(j_f/B = 15–20\) mA/Wb) and 0.3–0.5 \(\mu A/m^2\) when the IMF is northward (\(j_f/B = 7–10\) mA/Wb).

[31] The field lines studied here connect to latitudes of 68°–69.5° in the Kp = 1 version of the T89 magnetic field model. Field lines at \(|y| < 9R_E\) connect to the ionosphere between 2100 and 0300 LT. The typical latitude band covered by the field lines with high \(|j_f|\) is about 2°. Figure 1 of Weimer [2001] shows that the long-term-averaged ionospheric \(j_f\) at the foot points of the field lines studied here corresponding to \(|j_f/B| = 2–3\) mA/Wb when the IMF is northward and to 6 mA/Wb when the IMF is southward. The typical latitude width of these high current densities is 5° in Weimer’s [2001] model. This difference in latitude widths is associated with the fact that no ground station is always connected to an active plasma sheet field line. In contrast, the currents determined here used data only at times when the satellite was inside the plasma sheet, where Birkeland currents are concentrated. These latitude width differences suggest that the average current in a unit flux tube seen by a satellite while it is within the plasma sheet should be about 2–3 times the corresponding average current per unit flux seen at a fixed point above the auroral ionosphere, which is not always connected to field lines carrying large Birkeland currents. The currents calculated in this paper therefore are in reasonable agreement with Weimer’s [2001] model. We conclude that this study did not identify any significant difference between \(|j_f/B|\) at the PSBL and at the topside ionosphere on the \(|y| < 9R_E\) field lines.

[32] Although the decrease in \(|j_f/B|\) at \(|y| > 9R_E\) in the region that can be studied is real, this does not imply that the maximum Birkeland currents attained when flux tubes leave the CPS are small on field lines which connect to the flanks. As noted above, \(|j_f/B|\) is still increasing at \(x = -16R_E\) on the \(|y| > 9R_E\) field lines and the ability to use \(3 \times 3R_E\) boxes when data for all IMF directions were combined permitted this increase to be followed to \(x = -11.5R_E\). Our study of data sorted by IMF direction therefore did not determine the total Birkeland current on field lines which connect closer to the flanks. Weimer’s [2001] model suggests that \(|j_f/B|\) in the ionosphere continues to increase at local times beyond the region that could be studied here.

5. Summary, Discussion, and Conclusions

5.1. Summary of Techniques

[33] Particle and field measurements were sorted into boxes according to \(x\), \(y\), and \(\beta\), where \(\beta\) is given by
equation (2). This paper used $6 \times 6R_E$ $(x, y)$ boxes, eight $\beta_x$ boxes, and 4 years of Geotail data. The $x$ component of the momentum equation was used to calculate the average $z$ locations of the top and bottom of each $(x, y, \beta_y)$ box. All measurements then were interpolated to a fixed $z$ grid so that the derivatives in equation (4) could be taken and $j_\|z$ could be evaluated. Appendix A describes the methods that were used to estimate errors. It was concluded that only the Birkeland currents calculated at $x < -16R_E$ and $|z| > 1R_E$ were reliable.

5.2. MHD Processes

The results in this paper add to the model which has developed over the years describing the generation of long-term-averaged Birkeland currents and to the physics of magnetosphere-ionosphere coupling. The full sequence of interconnected events that result in $j_\|$ within the plasma sheet is extremely complex. A simplified summary of MHD mechanisms that have been shown to be particularly important for a study of the long-term-averaged $j_\|$ in the plasma sheet is shown in Figure 10 and discussed below. Three regions along a flux tube are distinguished in Figure 10. The plasma sheet, indicated by the subscript $p$, is the region examined directly using Geotail data. The region in which large parallel electric fields $E_\|$ sometimes form is referred to as the topside ionosphere. Below this is the conducting ionosphere, indicated by the subscript $i$.

The processes in Figure 10 are driven by the distribution of $\nabla \cdot P_{sw}$, the particle pressure, and effective viscous forces exerted by the solar wind on the magnetopause, and by the electromagnetic forces, which primarily depend on the IMF through its control of reconnection. These forces on the magnetopause are transmitted into and induce particle $\nabla \cdot P_p$ and electromagnetic $j_\| \times B_p$ forces in the plasma sheet. Any imbalance between these forces results in plasma acceleration and convection. If totally unbalanced, the $j_\| \times B_p$ force from a typical plasma sheet cross-tail current density $j_{py} = 1$ nA/m$^2$ and a typical neutral sheet $B_{pz} = 3$ nT would accelerate a plasma of density 0.3 particles/cm$^3$ from rest to a velocity of 700 km/s within a single $6 \times 6R_E$ box. Small-scale or short-lived events such as bursty bulk flows [Angelopoulos et al., 1992] require large accelerations and large imbal-
Figure 8. Similar to Figure 7 except for field lines on the dusk side ($y > 0$) of the plasma sheet.

Figure 9. Summary of the results from the $x = -16R_E$ boxes when the IMF $B_y < 0$. The $z = 0$ line is the neutral sheet where $B_z = 0$. Curves on which $B_y = 0$ and on which $j_\parallel = 0$ are identified.
iances between \( \mathbf{j}_p \times \mathbf{B}_p \) and \( \nabla \cdot \mathbf{P}_p \). In contrast, the long-term-averaged perpendicular flow velocities considered here change gradually, showing that on average \( \nabla \cdot \mathbf{P}_p \) and \( \mathbf{j}_p \times \mathbf{B}_p \) nearly balance and that inertial forces or plasma accelerations are small.

[36] The common electron and ion cross-field convection velocity \( \mathbf{V}_E \) at the neutral sheet, where \( B_x = 0 \) and \( B_y \) is small, generates a perpendicular polarization electric field at the neutral sheet \( \mathbf{E}_p \) with components given approximately by

\[
E_{px} = -B_{px}V_{Ey}; \quad E_{py} = B_{py}V_{Ex}.
\]

(6)

[37] Since \( \mathbf{V}_E \) is only the common portion of the electron and ion drift velocities it is difficult to measure directly, but can be estimated from the observations as described below.
Some or all of the potential drop associated with $E_p$, depending on whether an $E_i$ is or is not present above the conducting ionosphere is imposed on the ionosphere. When the ionospheric electric field $E_i$ is coupled with Ohm’s law and the Hall and Pedersen conductivities $\sigma_H$ and $\sigma_P$ it generates $j$, at this location in the ionosphere. Any divergence of the perpendicular part of $j$ requires $j_\parallel$ in the flux tube.

If a steady $j_\parallel$ which is demanded by the ionosphere exceeds what can be supplied by plasma sheet electrons in the loss cone, then an $E_i$ develops in the topside ionosphere, primarily in flux tubes carrying upgoing currents [Knight, 1973]. The resulting $E_i$ changes the electron distribution function by increasing the loss cone width, permitting plasma sheet electrons to carry more current to the ionosphere. The $E_i$ also reduces the $E_i$ imposed on the ionosphere, thereby reducing the $\mathbf{j}$, needed by Ohm’s law. Finally, since $E_i$ alters the precipitation of particles and $j$, alters the Joule heating, any changes in these parameters change $\sigma_H$ and $\sigma_P$.

5.3. Kinetic Processes

The above macroscopic picture provides a framework in which the results of this paper can be discussed. The MHD discussion emphasized the need for $j_\parallel$ above the ionosphere but did not show how current can be diverted from $j_\perp$ to $j_\parallel$ within the plasma sheet or how the plasma sheet can supply the electrons needed to sustain the average observed $j_\parallel$. It is only these plasma sheet processes that can be examined directly using Geotail data.

Birkeland currents are associated with parallel anisotropies in the electron and, to a small extent, the ion $f(v)$. One parallel anisotropy associated with $j_\parallel$ that has been observed at the ionospheric ends of field lines involves a relatively full loss cone in the downgoing $f(v)$ and a somewhat depleted loss cone in the upgoing $f(v)$. If upgoing particles from one hemisphere remained fully adiabatic as they passed through the plasma sheet and reached the opposite hemisphere there would also be a depleted downgoing loss cone. There then would be little difference between upgoing and downgoing fluxes, little net up-down anisotropy parallel to $\mathbf{B}$, and little $j_\parallel$. A process that depletes the loss cone particles at low altitudes and a separate process that fills the loss cone somewhere else both are needed to produce a long-term-averaged $j_\parallel$.

Loss cone filling is easy to understand for ions. Their orbits tend to be chaotic at the neutral sheet. Almost any ion $f(v)$ becomes nearly isotropic after having passed through the neutral sheet. Ions may also scatter through wave-particle interactions, but this scattering will be of little consequence if $f(v)$ is already isotropic. However, electrons carry almost all the observed $j_\parallel$ and electrons typically follow guiding center orbits throughout the neutral sheet. Some other electron scattering mechanism is therefore needed in conjunction with a loss cone to produce $j_\parallel$. Evidence of electron diffusion in the Geotail data was discussed in our previous study of perpendicular currents [Kaufmann et al., 2001]. Rapid electron scattering involves fluctuations with scale lengths on the order of a typical electron cyclotron orbit ($0.01R_E$) or periods comparable to the electron cyclotron frequency (10 Hz).

In addition to loss cones, source cone distributions such as beams and cones are observed at low altitudes. It again is necessary to have two processes to sustain $j_\parallel$, one that now enhances particle fluxes in the source cone at one end of a field line and another process that tends to isotropize $f(v)$ at some other point. To emphasize the need for two processes, Kaufmann et al. [2002] concluded that a parallel electric potential difference of approximately 200 V exists near the neutral sheet. This $E_i$ was needed to maintain charge neutrality in the region containing chaotic ion orbits and guiding center electron orbits. Electrons were accelerated in the parallel direction as they left the neutral sheet. However, since nearly all these electrons mirrored adiabatically far above the ionosphere they returned going in the opposite direction along the field line. The result of having an acceleration process near the neutral sheet but no scattering process near the mirror points is a bidirectional $f(v)$ with $T_\parallel > T_\perp$ but no net parallel electron flux and therefore no $j_\parallel$.

Frank et al. [1984], Paterson et al. [1998], and Mukai et al. [1998] showed that the electron and ion $f(v)$ can occasionally be measured accurately enough within the plasma sheet to directly determine $j_\parallel$. These cases all involved currents that are much larger than the long-term averages studied here. We were not able to use the available plasma sheet observations to directly measure the detailed asymmetries in $f(v)$ and the average $j_\parallel$.

5.4. Conclusions, Northward and Southward IMF

The buildup of $j_\parallel/B$ from zero near the neutral sheet to a maximum near the PSBL could be followed at $|y| < 9R_E$ using IMF northward or southward data and when data for all IMF directions were combined. It was found that the region 1 current per unit flux tube built up rather uniformly throughout the plasma sheet. In particular, current diversion was neither confined to the $|z| < 1R_E$ region in which ion orbits are chaotic nor to the PSBL. At larger $|y|$, $j_\parallel/B$ was still increasing at the edge of the region that could be studied. The preceding discussion suggests that the plasma sheet region in which $j_\parallel/B$ is building up is the region in which the isotropization process is taking place. A loss cone, source cone, or beam may be created at the ionospheric end of a flux tube. Any such structures would be removed by electron scattering in the plasma sheet region in which $j_\parallel/B$ is seen to build up.

A comparison with $j_\parallel/B$ in the ionosphere suggested that this ratio is nearly constant along a flux tube from the point at which it approaches the PSBL down to the region examined by the DE2 satellite which was used to create Weimer’s [2001] model. Peria et al. [2000] also found that $j_\parallel/B$ is relatively constant from the topside ionosphere to the lowest-altitude (300–1200 km) bin that was studied using FAST data. Therefore in practice only the ionosphere and the plasma sheet ends of a flux tube are important when studying the diversion of current from $j_\perp$ to $j_\parallel$.

The Birkeland currents determined using all data, using only data taken when the IMF was predominantly northward, and using only data for predominantly southward IMF all gave qualitatively similar patterns. The magnitude of $j_\parallel$ in the outer CPS was larger when the IMF was southward than when it was northward. It also was found that the average plasma sheet thickness was smaller.
and the average $|j_y|$ was larger on the dusk side when the IMF was southward. This may be attributed to the formation of very thin current sheets primarily on the dusk side in association with substorms or other geomagnetic activity. It also is possible that differences in the data distribution produced by orbital effects, as discussed in Appendix A, contributed to the dawn-dusk asymmetries.

The currents found here and in the same region of the magnetotail by Israelevich et al. [2001] were generally in the region 1 sense. Wing and Newell [2000] concluded that the sense of the current reversed near midnight in this region. This may indicate a problem with the magnetic field line projections. A number of studies [Frank et al., 1981; Elphic et al., 1985; Ohtani et al., 1988; Candidi et al., 1990] found region 0 currents at or slightly beyond the PSBL in the region studied. We could not study the PSBL because count rates became too low. It is possible that currents from some of these PSBL field lines may get mixed into data which, from the ionosphere, appears to project to closed middle tail plasma sheet field lines.

### 5.5. Type 1 Current System

Baumjohann et al. [1981] determined both $j_\perp$ and, through its divergence, $j_\parallel$ in the ionosphere for several auroral arc systems. Birkeland currents were found at the east and west ends of an auroral electrojet, and also in sheets at the northern and southern sides of the arcs. Comparable total perpendicular ionospheric currents flowed both in the north/south and east/west directions to connect these Birkeland currents.

Figure 11 is a sketch showing how a steady region 1 current which closes as an east–west ionospheric current can be maintained. This is referred to as a Boström type 1 current system [Boström, 1964]. In Figure 11 an entire block of plasma between the two field lines in the sketch is convecting earthward. A more realistic model could involve a superposition of many such systems. A dominant duskward $j_y$ throughout the tail-like region must be present to explain the reversal of $B_z$ between the northern and southern lobes. Both $B_z$ and the average common electron and ion convection velocity $V_{Ex}$ are observed to be predominantly positive near the neutral sheet earthward of $x = -25R_E$.

A plasma convecting earthward requires or will create a dawn to dusk polarization field $E_y$. This field builds up until $V_E = E \times B/B^2$. If there was no $j_\parallel$ and if $V_E$ did not change then there would be no net polarization current once a steady state was reached. However, if the projection of the polarization $E_y$ to the ionosphere produces a need for $j_y$ in the ionosphere, then $E_y$ can remain slightly smaller than the value given by equation (6). Electrons will continually be deflected westward and ions eastward to maintain $E_y$. This situation is similar to the laboratory Hall experiment except that both ions and electrons move at the same drift speed and contribute to the polarization field. The ionosphere is analogous to a detector which draws some current as the Hall electric field is being measured. In the magnetosphere the average $V_x$ and $B_x$ vary with distance from the Earth, but $E_y$ tends to remain more nearly uniform [Schödel et al., 2001].

Here we are particularly interested in transporting electrons across the tail to serve as the source of a steady $j_y$.

Polarization electrons can diffuse over substantial distances when they are being scattered, as appears necessary at least on the section of a field line where $|j_y/B|$ is changing. This polarization carries a steady flux of electrons to or away from the edges of the moving block of plasma. As shown in Figure 11 $j_{Hy}$, the total cross-tail current associated with
current diversion, cancels part of the dominant $j_i$, within the diversion region.

[53] The more energetic ions experience much faster gradient and curvature guiding center drifts than do the electrons. This difference in guiding center drift velocities produces cross-tail current in the direction opposite to the $j_i$ in Figure 11. Current diversion from this source involves a reduction of the guiding center drift current between the two flux tubes. Magnetization currents also contribute to the dominant duskward $j_i$, but do not transport electrons so are not considered as a potential source of a steady $j_i$. Any difference between contours of constant pressure and constant flux tube volume require $\nabla \cdot j_e$ and therefore $j_i$ [Wolf, 1983]. Our previous study of $j_i$ [Kaufmann et al., 2001] showed that electrons and ions carry comparable $j_i$ as seen in the Earth reference frame. Ions carry most of the $j_i$ on the dusk side of the tail, but ions drift almost earthward on the dawn side and therefore carry little $j_i$ here. The dominant electron $j_i$ on the dawn side could be attributed primarily to $E \times B$ drift in the presence of $E_x$ or $E_z$. Even though $E \times B$ drift produces no net ion plus electron current, it can explain the observed dawn side electron $j_i$. Scattering and cross-tail electron diffusion has been discussed here as an additional source of differential drift because electron scattering was associated with the current diversion process.

[55] A crude analysis of even the simple model in Figure 11 can help explain some aspects of the observations. Parallel currents are sometimes approximated using the linearized Knight’s [1973] relation

$$j_{ii} = K \phi_{ii},$$

where $K$ is constant for a given flux tube, $\phi_{ii}$ is the potential drop in the topside ionosphere, and $j_{ii}$ is the current density at the ionosphere. Lyons et al. [1979] analyzed rocket data to show that $K$ ranges from $1 \times 10^{-10}$ to $1 \times 10^{-9}$ A/m$^2$V and that the linear relation is often adequate in auroral arcs. Figure 11 has $j_{ii}$ flowing only in two unit flux tubes at the edges of the moving block of plasma. The total current in a unit flux tube $J_i = j_{ii} A_i$, is equal to $J_i$, the ionospheric current connecting the two flux tubes. The area $A_i$ of a unit flux tube at the ionosphere is $1/B_i$.

[55] The only other expression needed for the present study is Ohm’s law for the ionospheric current

$$J_i = \Sigma_i \phi_i/L_i$$

and the relation between potential drops

$$\phi_p = \phi_i + \phi_D,$$

where $\Sigma_i$ is an effective conductivity between the flux tube ends combining both Hall and Pedersen currents and effects of nonradial flux tube orientation, $L_i$ is the distance between the foot points of the two flux tubes, and $\phi_p$ and $\phi_i$ are potentials in the plasma sheet and ionosphere on the flux tube with the localized $E_i$. The potential is zero all along the other flux tube in the sketch. Equations 7–9 along with $j_{ii} = J_i$

$$J_i = \phi_i \left[ \frac{L_i}{\Sigma_i} + \frac{1}{KA_i} \right]^{-1},$$

where the $1/KA_i$ term arises from the presence of $E_i$. The effect of $E_i$ in the topside ionosphere can make the plasma sheet have different apparent properties as viewed from the conducting ionosphere. If $1/KA_i \gg L_i/\Sigma_i$ so that $E_i$ is large then $J_i \approx \phi_p/KA_i$. Since $J_i$ does not depend on $\Sigma_i$ in this limit, a given $V_{Es}$ or equivalently a given $\phi_p$ appears to impose a constant current source on the ionosphere. At the other limit $E_i$ is small so that $1/KA_i \ll L_i/\Sigma_i$ giving $J_i \approx \Sigma_i \phi_p/L_i$. In this limit $J_i$ is linearly proportional to $\Sigma_i$ and the ionosphere sees an apparent constant voltage source.

5.6. Type 2 Current System

[56] The sketch in Figure 12 shows how part of the Birkeland current measured in the plasma sheet can close as north–south currents in the conducting ionosphere. This is the Boström type 2 auroral current system [Boström, 1964]. Kaufmann et al. [2001] described a method to approximately separate the convection $V_E$ and the other drift $V_D$ components of the measured ion flow. The separation was not sufficiently reliable for quantitative studies. The paper concluded that $V_E$ had roughly equal components flowing away from midnight toward both the dawn and the dusk flanks as sketched in Figure 12. If this qualitative conclusion is correct, then polarization electric fields point-
ing earthward at dawn and tailward at dusk will be formed so that \( V_{Ey} = -E_y / B_z \) at the neutral sheet. When projected to the ionosphere the \( E_y \) in Figure 12 will drive a southward ionospheric current at dawn and a northward current at dusk in the Northern Hemisphere. Since the flowing plasma in the sketch has an inner boundary in the plasma sheet, the confined north–south \( E \) imposed on the ionosphere will require the parallel currents shown in Figure 12. As in Figure 11, the steady \( j_\parallel \) will be fed in the plasma sheet by polarization and other guiding center drift currents, here labeled \( J_{Tk} \). It is the diversion of electrons supplied by this \( J_{Tk} \) that permits a steady \( j_\parallel \) which is closed by north–south ionospheric currents.

[57] In any case, we conclude that the \( j_y \) measured here represents the diversion of perpendicular electron current to form parallel current throughout the plasma sheet. Scattering appears to be needed to maintain current continuity in the presence of steady electron loss in or extraction from the ionosphere.

### 5.7. Dawnward and Duskward IMF

[58] The plasma sheet exhibited clear symmetries when the IMF pointed northward or southward. At such times the plasma sheet \( B_z \) was positive throughout the dawn and southern dusk quadrants and negative in the other quadrants. It has been observed that the plasma sheet \( B_y \) tends to be negative at the neutral sheet when the IMF \( B_y < 0 \). This effect, which is sometimes referred to as leakage of the IMF \( B_z \) into the plasma sheet, is likely to be associated with twisting of the current layer [Lui, 1986; Kaymaz et al., 1994; Borovsky et al., 1998; Nishida et al., 1998; Tsyganenko et al., 1998; Kaufmann et al., 2000]. The addition of an extra negative contribution to \( B_y \) throughout the plasma sheet will shift the \( B_y = 0 \) contours in the sense shown in Figure 9. Since \( B_y \) is negative at the neutral sheet when the IMF \( B_y < 0 \), one has to move well southward of the neutral sheet at dusk and northward of the neutral sheet at dawn to reach points with \( B_y = 0 \). The \( B_y = 0 \) point represents the largest \( |y| \) reached by a given field line, and the \( B_y = 0 \) point is positioned in \(|y|\) for a field line. The separation between points at which \( B_y = 0 \) and at which \( B_y = 0 \) when the IMF points primarily dawnward or duskward is also evident in the data presented by Kaymaz et al. [1994].

[59] The tendency for \( j_y \) to be negative near \( z = 0 \) in Figures 7 and 8 also is easy to understand. Cross-tail current must be in the positive \( y \) direction on closed field lines in the tail-like geometry. This current system is required by Ampère’s law when \( B_z > 0 \) in the Northern and \( B_z < 0 \) in the Southern Hemispheres. When combined with the fact that the plasma sheet \( B_z \) tends to be negative for the IMF \( B_z < 0 \) situation, the dominant \( j_x B_x \) product must be negative.

[60] Plots made using only data with IMF \( B_x > 0 \) data were similar to Figures 7–9 with the patterns reflected about the \( y = 0 \) sheet and with the signs of \( j_x \) reversed. Plots made from the IMF \( B_y > 0 \) data set showed plasma sheet \( B_y > 0 \) and \( j_y > 0 \) near the neutral sheet. The regions with \( j_y < 0 \) were confined to large \( |z| \) in the northern dusk and southern dawn portions of the plasma sheet in the IMF \( B_y > 0 \) data set. The magnitudes of \( j_y \) were smaller when the IMF \( B_y \) was positive than when it was negative. Liou et al. [1998] found a similar effect in auroral power deposition. When IMF \( B_y > 0 \) and IMF \( B_x < 0 \) data were combined, most of the asymmetries shown in Figure 9 tended to cancel, producing patterns similar to those seen for northward and southward IMF in Figures 5 and 6. This is why Figure 4, which used data from all IMF orientations, is qualitatively similar to Figures 5 and 6.

[61] The sketch in Figure 13 illustrates the Northern Hemisphere field line distortions when the IMF \( B_y < 0 \). The dots at the ends of the field lines are at the neutral sheet. Since \( B_x = 0 \) and \( B_y < 0 \) at the neutral sheet, the \( x–y \) plane field line projections shown in Figure 13 point in the \( –y \) direction at their far ends. The field lines in Figure 13 point directly toward Earth at their Northern Hemisphere ionospheric ends, where the dipole field dominates. Between the neutral sheet and the ionosphere \( B \) is a combination of the field which would be present if the IMF \( B_y \) were zero plus the superimposed effect of this IMF \( B_y \). The plasma sheet magnetic field is seen in the sketch to have a negative \( B_y \) component all along the dusk side Northern Hemisphere field line. However, \( B_y \) must reverse from being negative at the neutral sheet to being positive in the ionosphere on the dawn side field line, as observed previously by Kaymaz et al. [1994]. The point at which \( B_y = 0 \) is indicated on the sketch and shown in Figure 9 at \( x = –16R_E \).

[62] To see why the \( j_x = 0 \) point is farther from the neutral sheet than the \( B_x = 0 \) point, it previously was noted that \( j_y > 0 \) near the dusk flank and \( j_x < 0 \) near the dawn flank [Israelievich et al., 2001; Kaufmann et al., 2002], as is shown in Figure 13. This \( j_x \) produces the observed flaring of magnetic field lines in the tail. The \( j \times B \) force associated with the neutral sheet \( j_x \) is toward midnight from both flanks when \( B_z > 0 \), thereby confining the higher pressure plasma that exists near midnight. The \( j_y \) and \( j_x \) in the plasma sheet retain the same signs for all IMF orientations because the tail is generally oriented along the solar wind flow direction. Since \( j_y < 0 \) on the dawn side, \( j \) must be perpendicular to the dawn side field line shown in Figure 13, or \( j_\parallel = 0 \), somewhere earthward of the point at which the plasma sheet \( B_y = 0 \).
The direction of the Earth’s B is reversed in the Southern Hemisphere, but \( B_y \) is still negative at the neutral sheet. It therefore is the dusk side field line that reverts direction or has a \( B_y = 0 \) point. Figure 9 includes the Southern Hemisphere observations for the IMF \( B_y < 0 \) case. Since the plasma sheet \( j_x > 0 \) on the dusk side, the point with \( j_y = 0 \) is again earthward of the point at which \( B_y = 0 \).

This separation of the \( B_y = 0 \), \( B_y > 0 \), and \( j_y = 0 \) surfaces therefore is a natural consequence of the superposition of a perturbation magnetic field that points predominately in the positive or negative \( y \) direction. It would be possible to define the neutral sheet as some location other than that at which \( B_y = 0 \), but this would not change the presence of distinct surfaces at which some measure of \( B \) and \( j_y \) are zero. In previous papers [Kaufmann et al., 2000, 2002] we tried defining the neutral sheet to be approximately the surface on which \( |B| \) is minimum. The criterion actually used for the neutral sheet was the surface of maximum ordinary plasma \( \beta \). The problem encountered was that \( \beta \) became very large only in very thin current sheets. Sorting by \( \beta \) therefore provided a good way to find thin current sheets, but looking at only large \( \beta \) conditions excluded the entire plasma sheet when \( \beta \) never exceeded whatever limit was selected. The present definition of \( B_y = 0 \), or the largest \( |x| \) reached along a closed flux tube, as the neutral sheet appears to be more appropriate for a study of typical properties of the tail.

Currents are in the region 1 sense, flowing down into the ionosphere on the dawn side and up on the dusk side in both hemispheres along all field lines as they leave the region investigated. Electrons appear to be scattered throughout the plasma sheet. This scattering removes the field-aligned anisotropies associated with loss cones, source cones, or other field-aligned asymmetries which were generated near the ionosphere. It is just that this point at which \( j_y = 0 \) is not coincident with the neutral sheet when the IMF points primarily downward or duskward. The Birkeland current per unit flux tube \( |j_y/B| \) becomes very large and changes rapidly near the neutral sheet for these IMF conditions. This suggests that electron scattering or parallel acceleration can be strong near the neutral sheet, where \( B \) is small. Figures 7–9 show that the limits on the region in which a reliable \( j_y \) could be calculated did not permit us to follow the buildup of \( |j_y/B| \) to its maximum level on field lines with \( j_y > 0 \) when the IMF \( B_y < 0 \).

**Appendix A: Uncertainties and Techniques**

**A1. Folded Data and Derivatives**

The procedure used near the neutral sheet whenever derivatives were needed was to carry out two separate analyses: one in which the data were and one in which data were not folded about the neutral sheet [Kaufmann et al., 2001]. Folding was done by reversing the signs of both \( B_x \) and \( B_y \) for all Southern Hemisphere (\( B_y < 0 \)) data points. A problem arises when taking derivatives using data that were folded in this manner. Folding places the neutral sheet at the edge of the data grid, and numerical derivatives become less reliable at the edge than within a gridded region.

It is for this reason that studies near the neutral sheet also were carried out keeping Northern and Southern Hemisphere data in separate boxes. These unfolded studies used four \( \beta \) boxes with \( B_x > 0 \) and four with \( B_x < 0 \) so the neutral sheet was at the center of the gridded region. Derivatives taken using unfolded data were compared with those taken using the folded data set. Substantial differences in \( j \) often were found in the box closest to the neutral sheet (Figures 2d and 2e of Kaufmann et al., 2001). For all other boxes the differences between the results using folded and unfolded data were comparable to the error estimates discussed in section 2.3. The resulting currents had similar magnitudes and were consistently in the region 1 sense in both hemispheres when unfolded data with no IMF selection were used. Figure 2 was prepared by averaging the results obtained from unfolded data in the two hemispheres for the first three \( \beta \) boxes. Derivatives obtained from the folded data were used for the other five \( \beta \) boxes.

**A2. Data Distribution**

The most important parameter determining uncertainties in the models is the number of independent data points per box. Even with \((x, y)\) boxes as large as \(6 \times 6 R_E\) there are 28 such boxes in the \(-31 < x < -7, |y| < 21 R_E\) gridding region. Each of these \((x, y)\) boxes is divided into \(8 \beta\) boxes giving a total of 224 \((x, y, \beta)\) boxes. The occasional use of \(3 \times 3 R_E\) \((x, y)\) boxes increases this number fourfold. Each box must contain a reasonable number of data points from enough different orbits to give representative averages of the parameters.

We carried out one complete study using only \( B_x > 0 \) data and another complete study using only \( B_x < 0 \) data. As would be expected, the calculated \( j_y \) was more erratic for each of these separate runs than for runs using all data. Orbital and seasonal effects create substantial differences between the number of available data points and therefore the accuracy of \( j_y \) calculations in the separate hemispheres. Geotail was near its 10\( R_E \) perigee in the tail at the summer solstice so there is much more plasma sheet data with \( B_x < 0 \) than with \( B_x > 0 \) near \( x = -10 R_E \). Geotail was near its 30\( R_E \) apogee in the tail at the winter solstice. There is more plasma sheet data with \( B_x > 0 \) than with \( B_x < 0 \) near \( x = -30 R_E \). The differences in the number of data points in the two hemispheres is substantially smaller at 30 than at 10\( R_E \) because tail flapping has a larger amplitude at larger radial distances. In the more distant region Geotail was often in the southern plasma sheet \((B_x < 0)\) when the GSM z satellite location was in the Northern Hemisphere. Even though the data distribution is not uniform, all runs showed the general features pointed out in Figure 2. The current was in the region 1 sense and the calculated \( |j_y| \) peaked near \( x = -16 R_E \), declining as one continues earthward along the field lines that reach large \( |z| \).

**A3. Uncertainties Near \( z = 0 \) and at \( x > -16 R_E \)**

Figure 3c shows results from the separate analyses for the set of field lines starting at \( y = -7.5 R_E \), one run using only \( B_x < 0 \) data and one run using only \( B_x > 0 \) data. Apogee has been placed at the center of the horizontal axis and perigee is at both left and right ends of the panel. For this particular \( y \) box the currents at the neutral sheet ends of most field lines almost appear to match for the separate \( B_x < 0 \) and \( B_x > 0 \) runs. Both runs show \( j_y < 0 \) at the neutral sheet endpoints between \( x = -20 \) and \( x = -10 R_E \). This matching is not consistently seen for other sets of field
lines. Figure 3d shows the same data that is displayed in Figure 3e except the horizontal axis is $z$ rather than $x$. It is difficult to match the individual $j_y$ curves in such a plot with the field lines shown in Figure 1.

[71] The $y = 7.5R_E$ dusk side panel in Figure 2 is an example of a set of field lines that exhibit unrealistic variations near the neutral sheet. This problem is particularly evident on field lines starting near or earthward of $x = -16R_E$. Figure 3e shows results from the calculations for this $y = 7.5R_E$ box using only $B_z > 0$ and using only $B_z < 0$ data. The fluctuations near the neutral sheet become most prominent in the $B_z > 0$ plot at small $|x|$. This is just the region in which orbital effects result in much less $B_z < 0$ data than $B_z < 0$ data as noted above. Similar increases in fluctuations near the neutral sheet are commonly seen in boxes for which there is little data, providing evidence that these variations are not real. Figure 3e shows that errors in $j_{\parallel}$ can be as large as 0.2 nA/m$^2$ when data coverage is poor.

[72] Figure 2 gives the misleading appearance that errors become small as one moves well away from the neutral sheet on the more distant field lines. The calculated $j_{\parallel}$ agrees so well for the various field lines only because $j_y$ is obtained at a given point by interpolating between $(x, y, z)$ boxes. The original $\beta_y$ boxes tend to be $1-2.5R_E$ thick near the PSBL, and Figure 1 shows that the selected field lines tend to be separated by $<1R_E$ in the $z$ direction when they approach the PSBL. It therefore is inevitable that interpolation will yield similar $j_y$ values on adjacent field lines. Likewise the shapes of the unphysical deflections near the neutral sheet for the $B_z > 0$ panel of Figure 3e are misleadingly similar. Here the field lines are $3R_E$ apart in the $x$ direction while an $(x, y)$ averaging box size of $6 \times 6R_E$ was used.

[73] Figures 3f and 3g show $j_{\parallel}/B$ based on data from the $y = \pm 7.5R_E$ panels in Figure 2. The abrupt changes near the neutral sheet are much more striking than the corresponding abrupt changes in Figure 2. This is because $B$ becomes small at the neutral sheet, and therefore the cross-sectional area of a unit flux tube becomes large. The reason errors became so large near $z = 0$ is related to our definition of the neutral sheet as the location where $B_z = 0$. This definition forces $j_yB_z$ to be very small within the first $\beta_y$ box. In contrast, $j_y$ is large in this box so a small error in the average $B_z$, produced, for example, by the crude aberration angle corrections used here, produces a large change in the calculated $j_y$. Since these sharp changes in $j_y$ can be eliminated or reversed by changing the aberrated $B_z$, by a small fraction of 1 nT, we conclude that the problem arises because of the methods used here. The $j_y$ calculated at $|z| < 1R_E$ has been discarded in Figures 4–8 due to the possibility of large relative errors.

[74] We also attribute the apparent peaking of $j_{\parallel}$ near $x = -16R_E$ (Figure 2) to difficulties in the calculations. Figures 3f and 3g illustrate how much more strongly peaked $j_{\parallel}/B$ is than is $|j_y|$. The cause of this artificial peaking involves the two most earthward $x$ boxes in the binning grid, which are centered at $x = -10$ and $-16R_E$ when $6 \times 6R_E$ $(x, y)$ boxes are used. Calculations of box thicknesses are based on the $x$ component of the momentum equation. The forces on one box are produced by $\nabla \cdot B$, by the gradient of the magnetic field pressure, and by the magnetic field line tension. The $x$ component of all these forces is well determined at $x = -16R_E$ because there are $x$ boxes both earthward and tailward of $-16R_E$. However, $x$ derivatives and the resulting $x$ component of magnetic field and particle pressure forces are less accurate for the most earthward box in the gridding region. As a result, the thicknesses calculated for the $x = -10R_E$ boxes are less reliable than elsewhere. This is the same problem that occurred when using folded data to take the $z$ derivatives needed to evaluate $j$ at the neutral sheet. Due to the orbital limits it is not possible to fix this problem at $x = -10R_E$ by extending the gridding region as was done by using unfolded data at the neutral sheet. This edge effect is not severe in the most tailward or $x = -28R_E$ box because parameters decrease nearly linearly in the distant tail. The problem is much worse, producing systematic errors at $x = -10R_E$ because the field is highly nonlinear as it makes the transition from dipolar to tail-like geometries.

[75] Some studies also were carried out using $3 \times 3R_E$ $(x, y)$ boxes. The results were erratic in some regions, but the $j_y$ patterns were relatively smooth on a number of field lines when data for all IMF orientations were combined. The most earthward $x$ box centers were at 8.5 and $11.5R_E$ when the $3 \times 3R_E$ grid was used. Plots of $j_y$ showed peaks at $11.5R_E$ with this grid spacing, supporting the conclusion that it is only the most earthward box that contains unreliable results.

[76] Another check that results are reliable in the $x = -16R_E$ boxes involved integrating $j_y$ from the neutral sheet to $|z|_{\text{max}} = 7R_E$ at each $(x, y)$ giving the total integrated current $J_y$ throughout the plasma sheet per unit distance in the $y$ direction. The calculated $J_y$ was compared to the results obtained by Tsyganenko et al. [1993] using substantially different methods and using data from a group of satellites that did not include Geotail. Our evaluations of $J_y$ for the box centered at $x = -16R_E$ agreed reasonably well with the results presented by Tsyganenko et al. [1993]. Both studies found a maximum $J_y$ of about $30 \text{kA}/R_E$ with the Tsyganenko et al. [1993] currents peaking near $|y| = 10R_E$. Our $J_y$ was largest in the box centered at $|y| = 6R_E$ and next largest in the adjacent box centered at $|y| = 12R_E$. Our parallel volume current density $j_y$ at $x = -16R_E$ was maximum in the $|z| = 4.5R_E$ box and decreased by about a factor of two in the outermost the $z$ box centered at $6.5R_E$. This shows that the regions containing the largest $j_y$ were included in the integration to $|z|_{\text{max}} = 7R_E$.

[77] In contrast, at $x = -10R_E$ the volume current density was largest in the outermost box centered at $|z| = 6.5R_E$. The cutoff of the $z$ gridding region therefore resulted in our missing a substantial fraction of the parallel current earthward of $x = -16R_E$. Tsyganenko et al. [1993] found a peak $J_y$ of $50 \text{kA}/R_E$ at $x = -10R_E$ while our integral to $|z| = 7R_E$ gave a peak of only $25 \text{kA}/R_E$. We conclude that our results are valid at $x < -16R_E$ but that the artificial decreases closer to Earth are produced by the techniques used here.

[78] A final test was used to identify errors in the calculations. Most points in the $\beta_y < 0.1$ boxes and a significant number of points in the $0.1 < \beta_y < 0.3$ boxes had to be discarded because of low count rates. This systematic deletion of a portion of the data contributes to uncertainties in the outermost $z$ boxes. An indication of this problem was seen as deviations from $\nabla \cdot B = 0$ in the average field data. The ratio $\nabla \cdot B \cdot \nabla \times B$ was calculated for each $(x, y, \beta_y)$ box. This ratio, which should be zero, was usually 0.1 or less when $\beta_y > 1$. The median value of this ratio exceeded 0.1
when $0.1 < \beta_1 < 1$ and sometimes became much larger. The largest ratios were seen in the $\beta_1 < 0.3$ boxes, suggesting that uncertainties maximize at the largest $|z|$ studied.

A4. Summary of Uncertainties

[79] The Birkeland current calculations were found to be unreliable in the box closest to the neutral sheet because of an extreme sensitivity to $B_z$ and therefore to aberration angle corrections. Beyond about $|z| = 1R_E$ the only effect seen when the aberration correction was varied was a small shift of the zero levels of the calculated parallel currents. Data in the $x = -10R_E$ boxes also were found to be unreliable. Figures 4–8 therefore only show results along the section of each field line.

A5. Incorporation of Solar Wind Data

[80] Solar wind and IMF data came from the Wind satellite. To project these measurements to Earth, it was assumed that solar plasma properties were uniform along Parker spiral contours. In the simplest approximation the resulting time delay between Wind and the center of the Earth is

$$\Delta t \approx \frac{x_w}{V_x} + \frac{y_w}{r_E\Omega_{ES}}, \quad (A1)$$

where $(x_w, y_w)$ is the location of the Wind satellite in GSE coordinates, $r_E$ is the distance from the Sun to the Earth, and $\Omega_{ES}$ is the rotation speed of the Sun as seen from the Earth ($2\pi/\Omega_{ES} \approx 27$ days). Data were used only when Wind was located at least $3R_E$ beyond the average shock location [Farris et al., 1991]. Use of equation (A1) makes it possible for the time order of points to differ at Wind and at Earth. Since timing corrections are not highly accurate and since we just used time shifts to the center of the Earth, the projected data were smoothed. All measurements that were calculated to arrive at Earth within $\pm 7.5$ min of the time of a Geotail measurement were averaged and used as the solar wind parameters and IMF for that particular time.

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References


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